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THEORETICAL AND PRACTICAL.

BY

WILLIAM G. PECK, LL. D.,

PROFESSOR OF MATHEMATICS AND ASTRONOMY IN COLUMBIA COLLEGE, AND
OF MECHANICS IN THE SCHOOL OF MINES.

A. S. BARNES & COMPANY,
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PREFACE.

THE object of the following pages is to present, within moderate limits, all the principles of arithmetic required by the student, the man of business, and the artisan.

To accomplish this end, the definitions and explanations have been revised and simplified, much superfluous matter has been rejected, and great care has been taken to avoid a multiplicity of cases and special rules, but in no instance has any essential principle been omitted, or abbreviated. It is believed that a careful examination of the work will show that all its parts are arranged in logical order; that the definitions and explanations are plain and concise; that the principles are stated clearly and accurately; that the demonstrations are full and complete; that the rules are perspicuous and comprehensive; and, finally, that the whole subject is amply illustrated by numerous graded examples and corresponding practical problems.

To meet the wants of a numerous class of teachers and pupils, the author has prepared a more elementary work, called the MANUAL OF PRACTICAL ARITHMETIC, from which all of the theory and some of the more abstruse principles have been omitted. This book is intended to precede the COMPLETE ARITHMETIC, but both are so constructed that either may be used without the other. The two books are arranged upon a common plan and the principal definitions and rules are essentially the same in both, but they differ widely in grade and in scope. The smaller one forms a natural introduction to the larger one, and the larger one is a fitting and appropriate sequel to the smaller one. If studied in succession, it is believed that better results will be reached than can be attained by devoting the same length of time to either separately.

NEW YORK, *May* 1, 1874.

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CHAPTER I.

DEFINITIONS AND PRINCIPLES.

Definitions.

1. A **Unit** is a single thing; as, *one pound, one foot, one day.*

2. A **Number** is a unit, or a collection of units; as, *one pound, three days, five feet.*

3. **Arithmetic** is the science of numbers.

It treats of the properties and relations of numbers, and of the operations that may be performed on them.

In what follows, the expressions 1°, 2°, 3°, etc., are read *first*, *second*, *third*, etc.

Formation of Numbers.

4.—1°. Numbers from *one* to *ten* are formed by collecting simple units, or *ones.*

A single unit is called *one;* one and one more are *two;* two and one more are *three;* three and one more are *four;* and so on, to *ten.*

2°. Numbers between *ten* and *one hundred* are formed by collecting *tens* and *ones.*

One ten and one are *eleven;* one ten and two are *twelve;* one ten and three are *thirteen;* and so on to two tens, or *twenty*. Two tens and one are *twenty-one*; two tens and two are *twenty-two*; and so on to three tens, or *thirty*. Four tens are *forty*; five tens are *fifty*; and so on to ten tens, or *one hundred*. The intermediate numbers between thirty and forty, forty and fifty, and so on, are formed in the same manner as those between twenty and thirty.

3°. Numbers between *one hundred* and *one thousand* are formed by collecting *hundreds*, *tens*, and *ones*; numbers between *one thousand* and *ten thousand* are formed by collecting *thousands*, *hundreds*, *tens*, and *ones*; and so on, indefinitely.

Numbers formed by collecting *ones*, in the manner just explained, are called *integers*; they are also called *integral*, or *whole numbers*.

Classification of Numbers.

5. Numbers are classified in two ways:

1°. They are either *abstract*, or *denominate*.

An **Abstract Number** is one whose unit is not named; as, *five*, *seven*, *eleven*.

A **Denominate Number** is one whose unit is named; as, *three pounds*, *six miles*, *seven months*.

2°. They are either *simple*, or *compound*.

A **Simple Number** is a collection of units of the same kind; as, *seven*, *eight yards*, *eleven ounces*.

A **Compound Number** is a collection of units of different kinds; as, *three feet and six inches*, *four hours and twenty minutes*, *six dollars and ten cents*.

Notation and Numeration.

6. **Notation** is the method of writing numbers by means of figures, or letters.

Numeration is the method of reading written numbers.

Figures.

7. The following figures are used in the *common*, or *decimal system of notation:*

0,	1,	2,	3,	4,	5,	6,	7,	8,	9.
naught,	**one,**	**two,**	**three,**	**four,**	**five,**	**six,**	**seven,**	**eight,**	**nine.**

These figures, taken separately, are called *digits*. The first one, named *naught*, is also called a *cipher*, or *zero;* it stands for *no number*. The remaining ones are called *significant figures;* they stand for the numbers written below them.

Figures are not numbers, but it will often be convenient to speak of them as such; in these cases, it is to be understood that we refer to the numbers which the figures represent.

Places of Figures.

8. If several figures are written in a line, the one on the right is said to stand in the *first place*, the one next to the right stands in the *second place*, the one next to it in the *third place*, and so on. Thus, in the expression 3784, 4 stands in the *first*, 8 in the *second*, 7 in the *third*, and 3 in the *fourth place*.

Orders of Units.

9. The number *one* is called *a unit of the first order;* the number *ten*, regarded as a collection of *ones*, is called *a unit of the second order; one hundred*, regarded as a collection of *tens*, is called *a unit of the third order;* and so on indefinitely, the unit of each succeeding order being *ten times* that of the next lower one.

The unit of the fourth order is *one thousand;* the unit of the fifth order is *ten thousand;* the unit of the sixth order is *one hundred thousand;* the unit of the seventh order is *one million;* and so on.

Units whose order is not named are supposed to be units of the first order, or *ones*.

General Principles of Notation and Numeration.

10. Numbers are written and read in accordance with the following principles:

1. *The same digit always represents the same number of units.*

2. *The order of units represented is denoted by the place in which the digit stands.*

3. *A cipher standing in any place shows that the number contains no units of that order.*

Thus, the expression 777 denotes 7 *hundreds*, 7 *tens*, and 7 *units*, that is, it stands for *seven hundred and seventy-seven*. In like manner, the expression 507 denotes 5 *hundreds*, 0 *tens*, and 7 *units*, that is, it stands for *five hundred and seven*. The expression 240 stands for *two hundred and forty*.

Places of figures and orders of units are counted from right to left, but numbers are written and read from left to right.

Examples in Notation and Numeration.

11. Any number less than one thousand may be written by the following

RULE.

Begin at the left and write the figures that denote the hundreds, tens, and units, in their proper order.

EXAMPLES.

Write the following numbers:

1. Fifty-seven. *Ans.* 57.
2. Ninety-four. *Ans.* 94.
3. One hundred and sixty-nine. *Ans.* 169.
4. Nine hundred and fourteen. *Ans.* 914.
5. Seven hundred and six. *Ans.* 706.
6. Eight hundred and sixty. *Ans.* 860.
7. Three hundred and sixty-five. *Ans.* 365.
8. Eight hundred and four. *Ans.* 804.
9. Two hundred and seventy. *Ans.* 270.
10. Nine hundred and eighteen. *Ans.* 918.
11. Seven hundred and seventy-seven. *Ans.* 777.
12. Nine hundred and ninety. *Ans.* 990.
13. One hundred and twelve. *Ans.* 112.
14. Six hundred. *Ans.* 600.
15. Three hundred and four. *Ans.* 304.
16. Four hundred and five. *Ans.* 405.
17. Six hundred and sixty-five. *Ans.* 665.
18. Three hundred and eight. *Ans.* 308.

Any written number less than one thousand may be read by the following

RULE.

Begin at the left and read the hundreds, tens, and units, in their order, translating figures into words.

EXAMPLES.

Read the following numbers:

1. 29. *Ans.* Twenty-nine.
2. 90. *Ans.* Ninety.
3. 118. *Ans.* One hundred and eighteen.
4. 506. *Ans.* Five hundred and six.
5. 670. *Ans.* Six hundred and seventy.
6. 977. *Ans.* Nine hundred and seventy-seven.
7. 835.
8. 270.
9. 809.
10. 422.
11. 109.
12. 186.
13. 204.
14. 309.
15. 470.
16. 999.

From what precedes, we see that *notation* is the operation of translating numbers from *words* into *figures*, and that *numeration* is the operation of translating numbers from *figures* into *words*.

The digits of a number indicate natural parts into which it may be separated. Thus, 986 may be separated into the three parts, 900, 80, and 6, each of which has its own unit. These parts are sometimes called *parts of numeration*.

Periods of Figures.

12. Written numbers, containing more than three figures, are separated into periods of three figures each, beginning at the right hand; the left hand period may contain less than three figures.

The first period, counting from the right, is called the *period of units;* the second is called the *period of thousands;* the third is called the *period of millions;* and so on, as shown in the following table, called

The Numeration Table.

Units of periods . .	Trillions,			Billions,			Millions,			Thousands,			Units.		
	hundreds of	tens of	units of	hundreds of	tens of	units of	hundreds of	tens of	units of	hundreds of	tens of	units of	hundreds of	tens of	units of
Periods	3	1	7,	8	3	2,	4	1	5,	8	1	6,	7	8	3.

The number written above is read, 317 *trillions,* 832 *billions,* 415 *millions,* 816 *thousands,* 783.

The unit of the first period is the simple unit *one;* the unit of the second period is *one thousand;* the unit of the third period is a thousand times one thousand, or *one million;* and so on, as indicated in the table, the unit of each period being equal to *a thousand times* that of the next lower one.

The table may be continued to any desired extent; the units of the succeeding periods are *quadrillions, quintillions, sextillions, septillions, octillions,* etc.

Every period, except the left hand one, must contain three figures, but they may all be ciphers. Periods that contain three figures are said to be *complete.*

Periods are written and read as explained in the last article. In writing, we make them all complete, *except* the one on the left; in reading, we name the unit of each, *except* the one on the right.

ADDITIONAL EXAMPLES.

13. Any number whatever may be written by the following

RULE.

Begin at the left and write each period in order, separating it from the following one by a comma.

Write the following numbers:

1. Twelve thousands, seven hundred and fifty-six. *Ans.* 12,756.

2. One hundred and fourteen thousands, eight hundred and seventy-nine. *Ans.* 114,879.

3. Seven hundred and fifty thousands, three hundred and eighty-nine. *Ans.* 750,389.

4. Nine hundred thousands, three hundred and fifty. *Ans.* 900,350.

5. Six millions, one hundred and sixty-nine thousands, four hundred and thirty-seven. *Ans.* 6,169,437.

6. Seventy-six millions, four hundred thousands, one hundred. *Ans.* 76,400,100.

7. 22 billions, 103 millions, 576 thousands, 102. *Ans.* 22,103,576,102.

8. 102 trillions, 125 millions, 403. *Ans.* 102,000,125,000,403.

9. 8 trillions, 7 billions, and 76. *Ans.* 8,007,000,000,076.

10. 12 millions, and 6. *Ans.* 12,000,006.

11. 16 millions, 3 thousand. *Ans.* 16,003,000.

Any written number may be read by the following

RULE.

I. Begin at the right and point it off into periods of three figures each; the left hand period may contain less than three figures.

II. Begin at the left and read the periods in their order, naming the unit of each, except that on the right.

EXAMPLES.

Read the following numbers:

1. 104,217.

Ans. One hundred and four thousands, two hundred and seventeen.

2. 2,304,516.

Ans. Two millions, three hundred and four thousands, five hundred and sixteen.

3. 1,001,010.

Ans. One million, one thousand, and ten.

4. 825,314,715.

Ans. 825 millions, 314 thousands, 715.

5. 376.
6. 2,141.
7. 13,350.
8. 716,717.
9. 1,810,003.
10. 21,800,002.
11. 16,000,200.
12. 18,742,000.
13. 6,003,021.
14. 785,003.
15. 2,303,492.
16. 7,723,326.
17. 8,543,420.
18. 217,330,795.
19. 60,000,300.
20. 500,500,500.

Roman Notation.

14. Roman Notation is the method of expressing numbers by letters. The letters used and the values they express are shown below:

Letters . . .	I,	V,	X,	L,	C,	D,	M.
Values . . .	1,	5,	10,	50,	100,	500,	1000.

Other numbers are expressed by combining these letters according to the following principles:

1°. *If a letter is repeated, the number that it denotes is repeated.*

2°. *If a letter that denotes a less number is written after one that denotes a greater number, the value of the latter is increased by that of the former.*

3°. *If a letter that denotes a less number is written before one that denotes a greater number, the value of the latter is diminished by that of the former.*

If a letter that denotes a less number is written between two that denote greater numbers, it diminishes the latter, but does not affect the former.

The method of applying these principles is shown in the following

TABLE.

I	denotes	1	XI	denotes	11	XXX	denotes	30	CCCC	denotes	400
II	"	2	XII	"	12	XL	"	40	D	"	500
III	"	3	XIII	"	13	L	"	50	DC	"	600
IV	"	4	XIV	"	14	LX	"	60	DCC	"	700
V	"	5	XV	"	15	LXX	"	70	DCCC	"	800
VI	"	6	XVI	"	16	LXXX	"	80	DCCCC	"	900
VII	"	7	XVII	"	17	XC	"	90	M	"	1000
VIII	"	8	XVIII	"	18	C	"	100	MM	"	2000
IX	"	9	XIX	"	19	CC	"	200	MDCCCLXXV denotes 1875.		
X	"	10	XX	"	20	CCC	"	300			

EXAMINATION QUESTIONS.

(1.) What is a unit? Example? **(2.)** What is a number? Example? **(3.)** What is arithmetic? What does it treat of? **(4.)** Explain the formation of numbers from *one* to *ten*; from *ten* to *one hundred*, etc. What is an integer, or whole number? **(5.)** In how many ways are numbers classified? What is an abstract number? A denominate number? A simple number? A compound number? **(6.)** Define notation. Numeration. **(7.)** Name the ten digits. What other names has the figure *naught?* Which are significant figures? **(8.)** Explain what is meant by the place of a figure. **(9.)** Explain what is meant by orders of units, and give the names of the orders up to the seventh. **(10.)** What are the three principles of decimal notation? How are places and orders counted, and how are numbers written and read? **(11.)** Give the rules for writing and reading any number less than 1000. **(12.)** What are periods of figures? Name the units of the first ten periods. **(13.)** Give the general rules for notation and numeration. **(14.)** What is Roman notation? Explain the method of writing numbers in this system.

CHAPTER II.

FUNDAMENTAL OPERATIONS.

I. ADDITION.

Definitions.

15. **Similar Numbers** are those which have a common unit. Thus, 3 *yards* and 7 *yards* are similar, but 3 *yards* and 7 *days* are not similar.

Abstract numbers are always similar, because they have the same unit.

16. The **Sum** of two or more numbers is a number that contains as many units as the given numbers taken together. Thus, 5 *days* is the sum of 3 *days* and 2 *days*.

17. **Addition** is the operation of finding the sum of two or more numbers.

The numbers added, and their sum, must be *similar*.

Explanation of Signs.

18. The *sign of addition*, +, is called *plus*; when placed between two numbers, it shows that the second is to be added to the first. Thus, the expression 4 + 5 shows that 5 is to be added to 4.

The *sign of equality*, =, indicates that the expressions between which it is placed are equal to each other. Thus, the expression $5 + 3 = 8$ indicates that the sum of 5 and 3 is equal to 8.

Principles of Addition.

19. The operation of addition depends on the following principles:

1°. *Any number is equal to the sum of all its parts.*

2°. *The sum of two or more numbers is equal to the sum of all their parts.*

3°. *The sum of the parts is the same, in whatever order the parts may be taken.*

Operation of Addition.

20. Let it be required to find the sum of 564, 783, and 688.

Explanation.—Having written the numbers so that units of the same order stand in the same column, we begin at the right and add each column separately. The sum of 8, 3, and 4, is 15 *units*, that is, 1 *ten* and 5 *units;* we write the 5 in the column of units and *carry forward* the 1 and add it to the column of tens. The sum of the tens, thus increased, $1 + 8 + 8 + 6$, is 23 *tens*, that is, 2 *hundreds* and 3 *tens;* we write the 3 in the column of tens and *carry forward* the 2 and add it to the column of hundreds. The sum of the hundreds, thus increased, $2 + 6 + 7 + 5$, is 20 *hundreds*, that is, 2 *thousands*, and 0 *hundreds;* as this is the last column, we set down the entire sum. The resulting number, 2,035, is the required sum, because it is the sum of all the *units*, *tens*, and *hundreds* of the given numbers (Art. **19**).

OPERATION.

$$\begin{array}{r} 564 \\ 783 \\ 688 \\ \hline 2035 \end{array}$$

In like manner other numbers may be added; hence, we have the following

RULE.

I. Write the numbers so that units of the same order shall stand in the same column.

II. Add the column of units; set down the simple units of the sum, and if there are any tens, carry them forward and add them to the next column.

III. Add the column of tens; set down the simple tens, and if there are any hundreds, carry them forward and add them to the next column.

IV. Continue this operation till all the columns have been added. Set down the entire sum of the last column.

EXAMPLES.

Perform the following additions:

	(1.)	(2.)	(3.)	(4.)
	315	29	215	8261
	423	814	27	3042
	719	302	891	171
Sum,	1457	1145	1133	11474

The rule holds good for all simple numbers, whether abstract or denominate.

(5.)	(6.)	(7.)	(8).
451 *feet.*	365 *days.*	187 *pounds.*	124 *things.*
817 *feet.*	821 *days.*	203 *pounds.*	287 *things.*
302 *feet.*	900 *days.*	866 *pounds.*	59 *things.*
917 *feet.*	76 *days.*	771 *pounds.*	803 *things.*
2487 *feet.*	2162 *days.*	2027 *pounds.*	1273 *things.*

The operation of adding a column of figures may be abbreviated by omitting the names of the figures added, and simply calling the results of the successive additions. Thus, in adding the first column in example 8th, instead of saying 3 and 9 are 12, and 7 are 19, and 4 are 23, we simply name the results, 12, 19, 23. In like manner, carrying 2 to the next column, we say 7, 15, 17 ; for the last column, we say 9, 11, 12.

(9.)	(10.)	(11.)	(12.)
9,102	8,760	25,678	87 *feet.*
479	325	3,002	236 *feet.*
73	512	21,001	1,443 *feet.*
810	786	715	2,010 *feet.*
4,312	1,420	1,630	7,818 *feet.*

(13.)	(14.)	(15.)	(16.)
62,743	27 *yards.*	7,478 *days.*	117,064
4,321	135 *yards.*	423 *days.*	92,973
78,731	7,271 *yards.*	79 *days.*	827,569
5,480	185 *yards.*	8,102 *days.*	1,351

Proof of Addition.—The proof of an operation consists in showing that it has been performed correctly.

1°. In addition, it is customary to add each column from *bottom* to *top*. To prove the work, we add each column from *top* to *bottom ;* the result should be the same as before.

Let each of the preceding examples be proved in this manner.

2°. Another method of proof consists in separating the numbers into two groups, adding each group separately, and then finding the sum of the results; this should be the same as that given by the rule, (Prin. 3°, Art. **19**).

Illustrations of the 2° Method.

$$\begin{array}{rrl}
 & \text{(17.)} & \textit{Proof.}\\
 & \left.\begin{array}{r}2387\\184\\376\end{array}\right\} & 2947\\
 & \left.\begin{array}{r}1325\\973\\2045\end{array}\right\} & \underline{4343}\\
\cline{2-2}
\text{Sum,} & 7290 & 7290
\end{array}
\qquad\qquad
\begin{array}{rl}
\text{(18.)} & \textit{Proof.}\\
\left.\begin{array}{r}23456\\78901\\23456\end{array}\right\} & 125813\\
\left.\begin{array}{r}78901\\23456\\78901\end{array}\right\} & \underline{181258}\\
\hline
307071 & 307071
\end{array}$$

In the following examples, let each result be proved by the last method:

19. Add 7,384; 326; 6,780; and 57. *Ans.* 14,547.

20. Add 6,740; 9,745; 5,769; 8,031; 6,543; 2,052; and 9,999. *Ans.* 48,879.

21. Add 89; 4,500; 423; 2,024; 5,408; 60,546; and 9,401.

22. Add 83,746 *yards*; 2,478 *yards*; 692,577 *yards*; 456 *yards*; and 7 *yards*.

23. Add 935,473 *dollars*; 262 *dollars*; 13,897 *dollars*; 598,453 *dollars*; 25 *dollars*; 3,734 *dollars*; and 72,405 *dollars*.

The sign $ written before a number signifies *dollars*; thus, the expression $120 is read 120 *dollars*.

24. Find the sum of $93,180; $279; $8,711; $371,809; $65; and $212,818.

25. Add 3,415; 17,382; 81,845; 162,345; and 8,312.

26. Add 8,492 *feet*; 14,592 *feet*; 112,897 *feet*; and 117,712 *feet*.

27. Add $8,842; $31,887; $113,214; and $887,319.

28. Find the sum of 385,842; 112,817; 32,413; and 33,335.

29. Find the sum of $88,141; $32,314; $141,003; and $89,947.

30. Add 114,312; 87,808; 3,214; 896; and 87.

31. Add 8,730; 3,021; 785; 879; and 92.

32. Add $87; $78; $114; $289; $176; and $95.

33. Add 42,314 *yards;* 119,342 *yards;* 8,962 *yards*; and 975 *yards*.

34. Add 17,439; 410,864; 842,317; 345,876; 79,884; and 18,719.

35. Add 714,312; 182,416; 312,867; 382,843; 79,816; and 43,115.

36. Find the sum of 3,345,816; 2,882,314; 387,892; 4,381,500; 2,874,316; and 887,342.

37. Add 188,841; 362,817; 411,217; 336,425; 814,316; and 45,554.

38. Add 214,333; 286,329; 851,426; 303,249; 12,456; 17,324; and 22,404.

39. Add 3,329,941; 187,693; 821,436; 2,227,438; 132,314; and 283,304.

40. Add 193,391; 4,180,280; 7,814,312; 88,430; 92,872; and 64,428.

41. Add 112,847; 186,320; 662,641; 3,400,300; 2,810,000; and 749,209.

42. Add 682,817; 336,336; 4,150,209; 2,390,374; and 86,810,304.

Dollars and cents may be written together, the cents being separated from the dollars by a point. Thus, the expression $17.84 is read 17 dollars and 84 cents.

Dollars and cents may be added like simple numbers. In performing the addition, the point that separates them is not to be considered.

(43.)	(44.)	(45.)	(46.)
$18.73	$5.83	$186.40	$413.30
23.47	10.19	75.75	325.15
15.62	27.03	37.18	414.82
7.91	11.94	201.92	97.45
112.13	203.07	184.42	111.32
684.21	211.46	36.35	202.16
73.19	305.24	41.15	113.27
19.06	802.41	72.27	814.42

PRACTICAL PROBLEMS.

21. A *problem* is a question proposed that requires an answer. The operation of finding the answer is called the *solution of the problem.*

Solve the following problems:

1. A farmer sold a span of horses for $318, two pairs of oxen for $420, and six cows for $290; how much did he receive? *Ans.* $1,028.

2. A man bought a house for $24,500, paid $1,675 for repairs, $3,140 for furniture, $375 for taxes, and then sold the whole for $2,155 more than the cost; what did he receive? *Ans.* $31,845.

ABBREVIATIONS.—In the following problems, *lbs.* stands for *pounds*; *ft.* for *feet*; *yds.* for *yards*; and *bu.* for *bushels*. Other abbreviations will be explained in their proper places.

3. A wagon is loaded with 5 boxes of goods; the first weighs 473 *lbs.*, the second 392 *lbs.*, the third 479 *lbs.*, the fourth 1,217 *lbs.*, and the fifth 376 *lbs.*; what is the weight of the entire load? *Ans.* 2,937 *lbs.*

4. The first car of a freight train contains 8,117 *lbs.*, the second 11,819 *lbs.*, the third 9,156 *lbs.*, the fourth 8,884 *lbs.*, the fifth 10,398 *lbs.*, and the sixth 9,982 *lbs.*; how many *lbs.* in all? *Ans.* 58,356 *lbs.*

5. A farm contains 79 *acres* of woodland, 63 of pasture, 50 of meadow, and 73 of arable land; how many acres in the farm? *Ans.* 265 *acres.*

6. A factory turned out 702 *yds.* of cloth on Monday, 1,023 *yds.* on Tuesday, 1,107 *yds.* on Wednesday, 997 *yds.* on Thursday, 910 *yds.* on Friday, and 1,045 *yds.* on Saturday; how many yards did it turn out in the week? *Ans.* 5,784 *yds.*

7. A merchant owes A. $2,160, B. $3,879, C. $813, D. $955, and E. $1,796; how much does he owe in all? *Ans.*. $9,603.

8. A farmer has 12 horses, 16 more oxen than horses, 42 more cows than horses and oxen together, and 22 more calves than oxen and cows together; how many in all? *Ans.* 254.

9. A gentleman built a house; the carpenter work cost him $4,285, the masonry $3,950, the plumbing and grates $2,183, the painting $1,975, and miscellaneous work $3,992; what was the entire cost?

10. A merchant buys 56,250 *bu.* of corn, 30,211 *bu.* of oats, 18,312 *bu.* of barley, 2,197 *bu.* of wheat, and 713 *bu.* of rye; how many bushels did he buy altogether?

11. The distance from Albany to New York is 144 *miles*, from New York to Philadelphia 90 *miles*, from Philadelphia to Baltimore 98 *miles*, from Baltimore to Washington 38 *miles*, and from Washington to Norfolk 217 *miles*; how far is it from Albany to Norfolk by this route?

12. In a lumber yard there are 37,412 *ft.* of spruce, 15,102 *ft.* of pine, 9,157 *ft.* of oak, 171,813 *ft.* of hemlock, 7,413 *ft.* of ash, and 18,002 *ft.* of chestnut: how many *ft.* of all kinds?

13. A work consists of 6 volumes: the first volume contains 611 pages, the second 539, the third 687, the fourth 599, the fifth 586, and the sixth 679: how many pages in the entire work?

14. A man bequeaths $15,750 to his daughter, $22,850 to each of two sons, and twice as much to his wife as to his daughter; what is the amount of his bequests?

15. The population of Maine is 627,413, of New Hampshire 301,471, of Vermont 300,187, of Massachusetts 1,240,499, of Connecticut 449,749, and of Rhode Island 192,815; what is the aggregate population of these States?

EXAMINATION QUESTIONS.

(15.) When are numbers similar? Illustrate. Why are abstract numbers similar? (16.) What is the sum of two numbers? Example? (17.) What is addition? What kind of numbers can be added? (18.) Explain the use of the sign plus, and the sign of equality. (19.) What are the principles of addition? (20.) Give the rule for adding simple numbers. What is meant by the proof of an operation? Give the methods of proving addition. (21.) What is a problem? What is the solution of a problem?

II. SUBTRACTION.

Definitions.

22. The **Difference** between two numbers is a number which, added to the *less*, will produce the *greater*. Thus, 6 is the difference between 10 and 4, because 4 + 6 = 10.

The greater number is called the *minuend*: the less number is called the *subtrahend*; and their difference is called the *remainder*.

23. Subtraction is the operation of finding the difference between two numbers.

The two numbers, and their difference, must be similar.

Explanation of Signs.

24. The *sign of subtraction*, –, is called *minus*; when placed between two numbers, it shows that the *second* is to be subtracted from the *first*. Thus, 5 – 3 shows that 3 is to be subtracted from 5.

A *parenthesis*, ( ), inclosing two or more numbers, shows that the inclosed expression is to be treated as a single number. Thus, 8 – (5 – 3) shows that the difference between 5 and 3 is to be subtracted from 8.

Principles of Subtraction.

25. The operation of subtraction depends on the following principles:

1°. *If all the parts of the subtrahend are taken from corresponding parts of the minuend, the sum of the partial remainders is equal to the required remainder.*

2°. *If the same number is added to both minuend and subtrahend, their difference is not changed.*

Operation of Subtraction.

26. Let it be required to find the difference between 565 and 393.

OPERATION.

Minuend,	565
Subtrahend,	393
Remainder,	172

Explanation.—We write the subtrahend under the minuend, so that units of the same order shall stand in the same column. Then, beginning at the right hand, we see that 3 *units* from 5 *units* leave 2 *units;* we therefore write 2 in the line below. Because 9 *tens* cannot be taken from 6 *tens*, we increase the latter by 10 *tens*, making it 16 *tens;* now 9 *tens* from 16 *tens* leaves 7 *tens;* we therefore write 7 in the line below. To compensate for the 10 *tens* added to the minuend, we add 10 *tens* to the subtrahend, (Principle 2°); but 10 *tens* is equal to 1 *hundred;* this we carry forward and add to the next figure on the left, which gives 4 *hundreds;* taking 4 *hundreds* from 5 *hundreds*, we have 1 *hundred*, which we write in the line below. The resulting number, 172, is the sum of the partial remainders obtained by subtracting the parts of the subtrahend from corresponding parts of the minuend; it is, therefore, from Principle 1°, the required remainder.

In like manner we may find the difference between any two numbers; hence, we have the following

RULE.

I. Write the less number under the greater, so that units of the same order shall stand in the same column.

II. Beginning at the right, subtract each figure in the lower line from the one above it, and write the difference in the line below.

III. If any figure in the lower line exceeds the one above it, increase the latter by 10, perform the subtraction, and then add 1 to the next figure in the lower line.

The operation described in the last clause of the preceding rule is called *carrying*. This operation, and that of adding 10, when required, are performed mentally.

EXAMPLES.

	(1.)	(2.)	(3.)	(4.)
From	663	976	704	1,806
Subtract	580	531	483	720
Remainder,	83	445	221	1,086

	(5.)	(6.)	(7.)	(8.)
From	4,236	80,502	46,095	555,555
Subtract	3,089	38,672	28,736	123,456
Remainder,	1,147	41,830	17,359	432,099

Proof.—Add the remainder to the subtrahend; if the sum is equal to the minuend, the work is correct.

ILLUSTRATIONS.

	(9.)	(10.)	(11.)
From	75,625	376,781	367,045
Subtract	24,319	95,845	106,253
Remainder,	51,306	280,936	260,792
Proof,	75,625	376,781	367,045

Let each of the following examples be proved:

12. From 8,416 subtract 2,918. *Ans.* 5,498.
13. From 30,811 subtract 13,240. *Ans.* 17,571.
14. From 27,880 subtract 9,226. *Ans.* 18,654.
15. From 35,846 subtract 12,829. *Ans.* 23,017.
16. From 75,901 subtract 17,980. *Ans.* 57,921.
17. From 37,229 subtract 17,991.
18. From 100,304 subtract 62,818.
19. From 1,000,302 subtract 888,772.
20. From 892,201 subtract 300,998.
21. From 1,000,000 subtract 333,333.

22. From 880,002 subtract 801,998.

23. From 1,000,892 subtract 1,395.

24. Find the difference between 516,315 and 211,209.

25. How much does 758,901 exceed 349,806?

26. What is the difference between 589,217,415 and 27,311,415?

27. If the sum of two numbers is 817,412, and one of the numbers is 212,494, what is the other?

28. The greater of two numbers is 30,011 and the less is 10,299; what is their difference?

29. The sum of two numbers is 485,752, and the less number is 82,992; what is the greater?

30. What is the difference between 40,690,980 and 699,690?

31. From 81,423 take 20,120.

32. From 80,200 subtract 1,875.

33. From 18,711 yards subtract 13,392 yards.

34. From 123,387 feet subtract 94,816 feet.

35. From $814,316 subtract $91,320.

36. From $620,306 subtract $413,314.

37. From $813,864 subtract $11,899.

38. From 111,336 yards subtract 77,814 yards.

39. Find the difference between $71,846,203 and $14,007,756.

40. From $4,496,802 take $183,846.

41. From 23,718,412 subtract 20,807,306.

42. From 887,892 subtract 709,278.

43. If the sum of two numbers is 68,823, and the less number is 29,394, what is the greater?

44. The greater of two numbers is 8,971,563, and the less is 4,043,989; what is their difference?

45. From 2,814,316 subtract 999,007.

46. What is the difference between 9,904,506 and 1,304,216?

47. Find the difference between 2,816,214 feet and 1,856,394 feet.

48. How much does 3,816,204 exceed 3,334,599?

49. What is the difference between 740,817 and 220,198?

50. From the sum of 862,141 and 32,843 subtract 884,109.

51. How much does the sum of 39,418 and 27,362 exceed the sum of 19,823 and 29,819?

52. Find the sum of 18,814 and 32,315, and subtract from it 17,794.

53. From the sum of $8,833, $141,209, and $11,362, subtract the sum of $2,843, and $10,906.

54. From the sum of 88,303 feet, and 67,119 feet subtract the sum of 74,395 feet, and 6,202 feet.

55. Diminish 688,043 by 342,890.

PRACTICAL PROBLEMS.

1. A. borrowed of B. $9,780 and paid $2,176; how much remained due? *Ans.* $7,604.

2. A. purchased a farm for $10,000 and paid thereon $4,790; how much remained due? *Ans.* $5,210.

3. B. bought merchandise, which he sold for $11,275, and made thereby $2,114; what was the cost price? *Ans.* $9,161.

4. In 1860 the population of Maine was 627,413, and in 1870 it was 913,279; what was the gain in 10 years? *Ans.* 285,866.

5. The sum of two numbers is 9,427, and the greater is 5,825; what is the less number?

6. In 1790 the population of Connecticut was 238,141, and in 1840 it was 309,978: what was the gain in that period?

7. In 1840 the population of Arkansas was 97,574, which was a gain of 67,186 in 10 years; what was the population of that State in 1830?

8. How much does 57,182 exceed 18,394?

9. A merchant commenced business with a capital of $21,308, and retired with $74,114: how much did he make?

10. A., B., and C. commence business; A. puts in $25,010, B. $11,700, and C. $20,150; at the end of a year they have together $149,711: how much did they gain?

11. A merchant bought 500 yards of linen for $276, 3,100 yards of muslin for $325, and 75 yards of broadcloth for $218, and sold the whole for $1,316; how much did he gain?

12. A. has a yearly income of $12,000; of this he spends for rent $2,750, for taxes, repairs, and insurance, $814, for clothing $1,342, for household expenses $4,211, and the remainder he distributes in charity; how much does he distribute?

13. B. has $12,244, and after paying his debts and giving away $3,108, he has remaining $8,199; what is the amount of his debts?

14. A merchant bought cloth for $1,592, silk for $1,274, lace for $816, and sold the cloth for $2,102, the silk for $1,190, and the lace for $969; how much did he gain?

15. A landholder owned 1,875 acres in Illinois, 2,396 acres in Indiana, and 13,742 acres in Michigan; of this he sold 813 acres in Illinois, 372 acres in Indiana, and 7,411 acres in Michigan; how many acres has he remaining?

16. A., B., and C. are in trade; A. gains $7,055, B. gains $813 less than A., and C. gains as much as A. and B. together, lacking $994; what do they all gain?

17. A. bought a farm for $8,192, expended $2,815 for improvements, paid $387 for taxes, and then sold it so as to lose $2,282; for what did he sell it?

18. A man worth $18,000 left $4,287 to his elder son, $3,754 to his younger son, $3,219 to his daughter, and the remainder to his wife; what was the wife's portion?

19. A man was 21 years old in 1843; in what year will he be 75 years old?

20. A merchant bought 4 cargoes of grain; the *first* contained 6,705*bu.*, the *second* contained 842*bu.* less than the first, the *third* contained 911*bu.* more than the second, and the *fourth* contained 3,092*bu.* less than the second and third together; how many *bu.* were there in the four cargoes?

21. A man bought three estates: for the *first* he gave $5,260, for the *second* he gave $3,585, and for the *third* he gave as much as for the first two together; he afterward sold them all for $15,280; did he gain or lose, and how much?

EXAMINATION QUESTIONS.

(22.) What is the difference between two numbers? Illustrate. What is the minuend? The subtrahend? The remainder? (23.) What is subtraction? (24.) What is the name and use of the sign of subtraction? What is the use of the parenthesis? (25.) What are the principles of subtraction? (26.) Give the rule for subtraction. How is subtraction proved?

III. MULTIPLICATION.

Definitions.

27. Multiplication is the operation of taking one number as many times as there are units in another.

The first number, that is, the number taken, is called the *multiplicand*; the second number is called the *multiplier*; and the result is called the *product*. Thus, 4 taken 3 times is equal to 4 + 4 + 4, or to 12; in this case 4 is the multiplicand, 3 is the multiplier, and 12 is their product.

Both *multiplicand* and *multiplier* are called *factors* of the product. Thus, 4 and 3 are factors of 12.

Sign of Multiplication.

28. The *sign of multiplication*, ×, placed between two numbers indicates that their product is to be taken. Thus, 4 × 3 denotes the product of 4 and 3.

The value of the product does not depend on the order of its factors. Thus, 4 × 3 is the same as 3 × 4.

29. The operation of multiplication requires a thorough knowledge of the following table, called the

Multiplication Table.

1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 1
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36
5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8
5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16
5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24
5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32
5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40
5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48
5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56
5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64
5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72
9 × 1 = 9	10 × 1 = 10	11 × 1 = 11	12 × 1 = 12
9 × 2 = 18	10 × 2 = 20	11 × 2 = 22	12 × 2 = 24
9 × 3 = 27	10 × 3 = 30	11 × 3 = 33	12 × 3 = 36
9 × 4 = 36	10 × 4 = 40	11 × 4 = 44	12 × 4 = 48
9 × 5 = 45	10 × 5 = 50	11 × 5 = 55	12 × 5 = 60
9 × 6 = 54	10 × 6 = 60	11 × 6 = 66	12 × 6 = 72
9 × 7 = 63	10 × 7 = 70	11 × 7 = 77	12 × 7 = 84
9 × 8 = 72	10 × 8 = 80	11 × 8 = 88	12 × 8 = 96
9 × 9 = 81	10 × 9 = 90	11 × 9 = 99	12 × 9 = 108

The combinations in this table are expressed by signs, and may be read in two different ways. Thus, the combination 6 × 8 = 48 may be read, 6 *times* 8 *are* 48, or it may be read, 6 *eights are* 48.

Effect of Annexing Ciphers.

30. Every cipher that we annex to a number moves each of its digits one place to the left, that is, it converts *units* into *tens*, *tens* into *hundreds*, and so on; but this is the same as multiplying the number by 10; hence,

To multiply a number by 10, we annex *one cipher*; to multiply it by 100, we annex *two* ciphers; to multiply it by 1000, we annex *three* ciphers; and so on. Thus, 75 × 10 = 750; 75 × 100 = 7,500; 75 × 1000 = 75,000.

To multiply by *any number of tens*, we first multiply by the *given number* and then annex *one* cipher; to multiply by *any number of hundreds*, we multiply by the *given number* and annex *two* ciphers; to multiply by *any number of thousands*, we multiply by the *given number* and annex *three* ciphers; and so on. Thus, 8 × 4 *tens* = 32 *tens* = 320; 6 × 4 *hundreds* = 24 *hundreds* = 2,400; 7 × 3,000 = 21,000.

Principles of Multiplication.

31. The operation of multiplication depends on the principles just explained and also on the following:

If all the parts of the multiplicand are multiplied by each part of the multiplier, the sum of the partial products is equal to the required product.

Operation of Multiplication.

32. Let it be required to find the product of [illegible] and 346.

OPERATION.

Multiplicand,	458
Multiplier,	346
Partial products,	2748 1832 1374
Product,	158468

Explanation.—Having written the numbers so that units of the same order stand in the same column, we begin at the right and multiply all the parts of the multiplicand by 6. Multiplying 8 *units* by 6, we have 48 *units*, or 4 *tens* and 8 *units;* we write 8 in the column of units and carry 4 to the column of tens; multiplying 5 *tens* by 6, we have 30 *tens*, which, increased by the 4 *tens* brought forward, gives 34 *tens*, or 3 *hundreds* and 4 *tens;* we write 4 in the column of tens and carry 3 to the column of hundreds; multiplying 4 *hundreds* by 6, we have 24 *hundreds;* this, increased by the 3 *hundreds* brought forward, gives 27 *hundreds*, or 2 *thousands* and 7 *hundreds*, which we write down.

We next multiply all the parts of the multiplicand by 4 *tens*, or 40. Multiplying 8 *units* by 40 (Art. 39), we have 320, that is, 3 *hundreds* and 2 *tens;* we write the 2 in the column of tens and carry the 3 to the column of hundreds, and so on. We next multiply all the parts of the multiplicand by 2 *hundreds.* Multiplying 8 *units* by 300 we have 2400, or 2 *thousands* and 4 *hundreds;* we write 4 in the column of hundreds and carry 2 to the column of thousands; and so on as before.

The sum of the products thus obtained is 158,468; but this is *the sum of the partial products found by multiplying all the parts of the multiplicand by each part of the multiplier;* it is therefore the required product, (Art. 31).

In like manner we may find the product of any two numbers; hence, the following

RULE.

I. Write the multiplier under the multiplicand, so that units of the same order shall stand in the same column.

II. Beginning at the right, multiply all the parts of the multiplicand by each figure of the multiplier, writing the first figure of each partial product under the corresponding multiplier.

III. Find the sum of the partial products.

In forming partial products, we set down and carry as in addition.

The multiplier must be abstract, but the multiplicand may be either abstract or denominate.

The product is always similar to the multiplicand.

EXAMPLES.

Perform the following multiplications:

	(1.)	(2.)	(3.)	(4.)
Multiplicand,	357	8645	2079	84123
Multiplier,	5	8	9	6
Product,	1785	69160	18711	504738

(5.)	(6.)	(7.)	(8.)
8842 *yds*	3749 *in.*	13146 *lbs.*	$81386
4	7	9	8
35368 *yds.*	26243 *in.*	118314 *lbs.*	$651088

We may multiply by 10, 11, or 12, in the same manner as by a single figure.

(9.)	(10.)	(11.)	(12.)
876	2345	998	8134
10	12	11	12
8760	28140	10978	97608

(13.)	(14.)	(15.)	(16.)
7312 *yds.*	8183 *lbs.*	18889 *ft.*	$18476
11	12	12	11
80432 *yds.*	98196 *lbs.*	226668 *ft.*	$203236

(17.)	(18.)	(19.)	(20.)
843	1817	7287	325
27	69	75	503
5901	16353	36435	975
1686	10902	51009	1625
22761	125373	546525	163475

Proof.—Multiply the multiplier by the multiplicand; if the product is the same as before, the work is correct.

(21.)	*Proof.*	(22.)	*Proof.*
345	572	835	794
572	345	794	835
690	2860	3340	3970
2415	2288	7515	2382
1725	1716	5845	6352
197340	197340	662990	662990

Let the following examples be proved by the above method:

23. Multiply 875 by 349.
24. Multiply $1,843 by 216.
25. Multiply 1,781 by 74.
26. Multiply 999 by 777.
27. Find the product of 1,754 and 306.
28. Find the product of 7,506 and 45.
29. What is the product of 206 and 1,008?
30. What is the product of 8,435 and 71?

When the multiplier terminates in ciphers, multiply the significant part, and annex ciphers as explained in Article 30.

(31.)	(32.)	(33.)	(34.)
31	875	42	87
290	4300	31000	1500
8990	3762500	1302000	130500

35. Find the product of 87 and 78,000.
36. Multiply 314 by 87,000.
37. What is the product of 414 and 82,000?

If both multiplicand and multiplier terminate in ciphers, multiply the significant parts, and annex as many ciphers as there are in both factors.

38. Multiply 8,840 by 7,250.
39. Multiply 2,040 by 8,060.
40. What is the product of 10,800 and 870?
41. Find the product of 8,020 and 870.
42. Multiply 37,300 by 8,170.
43. What is the product of 8,833 and 37,000?
44. Multiply 377 by the product of 29 and 8.
45. Find the product of 896 by 342.
46. Multiply 1,843 by 432.
47. Multiply 7,406 by 324.
48. What is the product of 3,889 and 643?
49. Find the product of 8,942 by 876.
50. Multiply 88,324 by 775.
51. Multiply 26,314 by 394.
52. Multiply 46,343 by 437.
53. Multiply 18,826 yards by 415.
54. Multiply 463,821 by 345.
55. Find the product of 84,802 and 176.
56. Multiply $24,367 by 1,292.
57. Multiply 18,426 by 1,384.
58. What is the product of 7,883 and 828.
59. Multiply $14,442 by 1,195.

Additional Definitions.

33. A *continued product* is a product of more than two factors. Thus, $3 \times 4 \times 5$ is a continued product: it indicates that the product of 3 and 4 is to be multiplied by 5.

A continued product may contain any number of factors. Its value is independent of the order of the factors. Thus, $3 \times 4 \times 5 = 3 \times 5 \times 4 = 4 \times 3 \times 5$.

34. A *composite number* is a number that is composed of two or more integral factors. Thus, 21 is a composite number, because it is equal to 3×7; the number 30 is composite, because it is equal to $2 \times 3 \times 5$.

To multiply by a composite number, we may multiply by each of its factors in succession.

EXAMPLES.

1. Find the product of 873 and 144.

The factors of 144 are 12 and 12.

OPERATION

873	
12	
10476	**Partial product.**
12	
125,712	**Total product.**

If the two factors are unequal, we generally multiply by the larger one first.

2. Find the product of 887*ft.* by 84.

The factors of 84 are 12 and 7.

3. Multiply $3,845 by 63.

The factors of 63 are 9 and 7.

4. Multiply 38,257*yds.* by 96.

5. Multiply 7,836*lbs.* by 132.

6. What is the product of 736 and 48?

In forming the continued product of several factors, it is better to commence with the larger ones.

7. Find the continued product of 17, 13, 11, and 5.

8. Find the continued product of 85, 42, 12, and 7.

PRACTICAL PROBLEMS.

In applying the rule for multiplication to practical problems, we multiply as though both factors were abstract, and then determine the unit of the product from the nature of the question.

ILLUSTRATION.

1. What will 455*lbs.* of sugar cost at 14 cents per pound?

Solution.—Since 455*lb.* will cost 455 times as much as 1*lb.*, it will cost 455 times 14 *cents*, that is, the number of cents in the answer is equal to the product of 455 and 14; finding this product, and annexing the name of the unit, we have,

Ans. 6,370 cents.

2. What will 692 pounds of beef cost at 26 cents per pound? *Ans.* 17,992 cents.

3. If one barrel of pork costs $15, what will 1,728 barrels cost? *Ans.* $25,920.

4. If a train travels 35 miles per hour, how far will it travel in 425 hours? *Ans.* 14,875 miles.

5. If it requires 720 barrels of provisions to feed an army for one day, how many barrels would be required for 365 days? *Ans.* 262,800 barrels.

6. An army contains 106 regiments, and each regiment contains 1,128 men; how many men in the army?

7. There are 320 rods in a mile; how many rods are there in 50 miles?

8. If a railway costs $42,500 per mile, and is 385 miles long, how much will it cost?

9. A field containing 56 acres produces 29*bu.* of rye to the acre; what is the total yield?

10. Sound travels at the rate of 1,142 feet per second; how far will it travel 3,600 seconds, or one hour?

11. The distance from New York to Bridgeport is 56 miles; allowing 320 rods to the mile, how many rods between the two cities?

12. In an orchard there are 214 rows of trees, and each row contains 241 trees; how many trees are there in the orchard?

13. What is the continued product of 92, 37, and 45?

14. A freight train consists of 21 cars; each car contains 85 barrels of flour, and each barrel of flour weighs 196 pounds; how many pounds in the entire cargo?

15. The distance from Bridgeport to New Haven is 18 miles; each mile contains 1,760 yards, and each yard 3 feet: how many feet from one city to the other?

16. In an orchard there are 14 rows of peach trees: each row contains 27 trees, and each tree bears 108 peaches; how many peaches in the orchard?

17. Find the continued product of 70, 20, and 80.

18. What is the continued product of 75, 50, and 100?

EXAMINATION QUESTIONS.

(27.) What is multiplication? Multiplicand? Multiplier? Product? Define factors. (28.) Explain the use of the sign of multiplication. (30.) How do you multiply a number by 10? By 100? By 1000? By any number of tens? (31.) What is the fundamental principle of multiplication? (32.) Give the rule for multiplication. If the multiplicand is denominate, what will be the nature of the product? What is the method of proving multiplication? What is the rule for multiplying when both factors terminate in ciphers? (33.) What is a continued product? How many factors may such a product contain? (34.) What is a composite number? How do you multiply by a composite number?

IV. DIVISION.

Definitions.

35. Division is the operation of finding how many times one number is contained in another.

The number to be divided is called the *dividend*; the number by which it is divided is called the *divisor*; the result of the division is called the *quotient*; and the part of the dividend that remains after the operation is called the *remainder*.

When the remainder is 0, the division is said to be *exact*; in in this case both *the divisor* and *the quotient* are *factors* of the dividend.

Methods of Indicating Division.

36–1°. The sign of division, ÷, when placed between two numbers, indicates that the *first* is to be divided by the *second*. Thus, the expression 12 ÷ 3 indicates that 12 is to be divided by 3.

2°. The same operation may be indicated by writing the dividend over the divisor, with a line between them.

Thus, the expression $\frac{12}{3}$ is equivalent to $12 \div 3$; it is read 12 *divided by* 3.

3°. Division may also be indicated by writing the divisor on the left of the dividend, with a line between them. Thus, the expression 3)12 indicates that 12 is to be divided by 3.

Object of Division.

37. Division is the *reverse* of multiplication. In multiplication, we have two factors given, to find their product; in division, we have the product and one factor given, to find the other factor.

The *multiplication table* may be used as a *division table* by reading the combinations in a reverse order. Thus, the combination $8 \times 9 = 72$, taken in a reverse order, may be read *the quotient of* 72 *by* 9 *is* 8.

Principles of Division.

38. The operation of division depends on the principles obtained by reversing those of Article **30**, and also on the following, obtained by reversing that of Article **31**:

If we divide all the parts of the dividend by the divisor, the sum of the partial quotients is equal to the required quotient.

39. There are two cases: 1°. *Short division*, in which the divisor contains but one figure; and, 2°. *Long division*, in which the divisor contains more than one figure.

In the *first* case most of the operation is performed mentally; in the second case the different steps of the operation are written out. The principles employed are the same in both cases.

1° Case. Short Division.

40. Let it be required to divide 26,812 by 4:

OPERATION.

$$\begin{array}{lr} & \text{Dividend.} \\ \text{Divisor,} & 4\,)\,\underline{26812} \\ \text{Quotient,} & 6703 \end{array}$$

Explanation.—Having written the numbers as shown in the margin, we begin at the left and divide the different parts of the dividend by the divisor.

Since 2 is not divisible by 4, we divide 26 by 4; this gives 6 for a quotient with 2 for a remainder; hence, there are 6 *thousands* in the quotient; we write 6 in the column of thousands and to the remainder we annex the following figure of the dividend giving 28 *hundreds*. The quotient of 28 by 4 is 7; hence, there are 7 *hundreds* in the quotient; we therefore write 7 in the column of hundreds. Since there is no remainder and since 1 is smaller than 4 there are no *tens* in the quotient; we therefore write 0 in the place of tens and annex the following figure to 1 giving 12 *units*. The quotient of 12 by 4 is 3, which we write in the column of units.

In this operation the dividend has been separated into parts, each of which is divisible by 4, as shown below:

$$\text{Parts,}\quad \left\{\begin{array}{r} 24000 \div 4 = 6000 \\ 2800 \div 4 = 700 \\ 12 \div 4 = 3 \end{array}\right\}\quad \text{Partial quotients.}$$

$$26812 \div 4 = 6703 \qquad \text{Quotient.}$$

The number 6703 is the sum of the quotients of all these parts by 4; it is therefore the required quotient (Art. 38).

In like manner we may treat all similar cases; hence, the following

RULE.

I. Write the divisor on the left of the dividend, and draw a line between them.

II. Divide the first figure of the dividend by the divisor and set the quotient underneath, or, *if the first figure is less than the divisor, divide the first two figures and set the quotient under the second.*

III. To the remainder annex the following figure of the dividend, divide the result by the divisor and set the quotient underneath, or, if the result is less than the divisor, put a cipher in the quotient, annex another figure, and proceed as before.

IV. Continue the operation till all the figures of the quotient have been found.

EXAMPLES.

Perform the following divisions:

	(1.)	(2.)	(3.)	(4.)	(5.)
	5)785	6)804	8)1624	7)392	9)1926
Ans.	157	134	203	56	214

	(6.)	(7.)	(8.)	(9.)	(10.)
	4)1544	3)825	8)4896	9)792	7)2415
Ans.	386	275	612	88	345

If there is a remainder after the last partial division, we write it over the divisor and annex the result to the quotient. Thus, $27 \div 4 = 6\frac{3}{4}$ indicates that the quotient of 27 by 4 is 6 with a remainder 3. The expression $6\frac{3}{4}$ may be read 6 and 3 *divided by* 4, or 6 and 3 *fourths.* Expressions of the form $\frac{3}{4}$ are called fractions: their nature and signification will be explained hereafter.

	(11.)	(12.)	(13.)	(14.)
	5)176	4)8140	7)8146	9)4023
Ans.	$35\frac{1}{5}$	2035	$1163\frac{5}{7}$	447

PROOF.—Multiply the quotient by the divisor and to the product add the remainder; if the result is equal to the dividend, the work is correct. Thus, in Example (11), we have, 35 × 5 + 1 = 176; hence, the work is correct.

Perform the following divisions and prove the work in each case:

15. Divide 12,360 by 4. *Ans.* 3,090.
16. Divide 3,730 by 5. *Ans.* 746.
17. Divide 20,202 by 6. *Ans.* 3,367.
18. Divide 37,904 by 4. *Ans.* 9,476.
19. Divide 90,872 by 8. *Ans.* 11,359.
20. Divide 640,339 by 7. *Ans.* 91,477.

We may divide by 10, 11, or 12, in the same manner as by a single figure.

(21.)	(22.)	(23.)	(24.)
10)8760	11)10978	12)8134	11)203236
876	998	$677\frac{10}{12}$	18476

25. Divide 837 by 7.
26. Divide 1,117 by 8.
27. Divide 3,123 by 3.
28. Divide 8,184 by 8.
29. Divide 18,765 by 9.
30. Divide 8,845 by 5.

31. Divide 78,886 by 12.
32. Divide 77,077 by 11.
33. Divide 6,809 by 11.
34. Divide 23,736 by 12.
35. Divide 154,824 by 12.
36. Divide 14,784 by 11.

2° Case. Long Division.

41. Let it be required to divide 60,198 by 237.

OPERATION.

Divisor. Dividend. Quotient.

$$\begin{array}{lr} & 237)60198(254 \\ & \underline{474} \\ & 1279 \\ & \underline{1185} \\ & 948 \\ & \underline{948} \\ \textbf{Remainder,} & 0 \end{array}$$

Explanation.—Having written the divisor on the left, we divide the *parts* of the dividend by it, as in short division.

Since 60 is not divisible by 237, there are *no thousands* in the quotient; annexing 1, we have 601 *hundreds.* The quotient of 601 by 237 is greater than 2, but less than 3; hence, there are 2 *hundreds* in the quotient; multiplying 237 by 2 *hundreds*, we have 474 *hundreds*, and this taken from 601 *hundreds*, leaves 127 *hundreds*, to which we bring down and annex the 9 *tens*, giving 1,279 *tens*. The quotient of 1,279 by 237 is greater than 5 and less than 6; hence, there are 5 *tens* in the quotient; multiplying 237 by 5 *tens*, we have 1,185 *tens*, and this taken from 1,279 *tens*, leaves 94 *tens*, to which we bring down and annex the 8 *units*, giving 948. The quotient of 948 by 237 is 4; hence, there are 4 *units* in the quotient; multiplying 237 by 4, we have 948, which taken from 948, leaves 0.

In this operation we have separated the dividend into parts, each of which is divisible by 237, as shown below:

$$\begin{array}{lrl} \textbf{Parts,} & \left\{\begin{array}{r} 47{,}400 \div 237 = 200 \\ 11{,}850 \div 237 = 50 \\ 948 \div 237 = 4 \end{array}\right\} & \textbf{Partial quotients.} \\ \textbf{Dividend,} & \overline{60{,}198 \div 237 = 254} & \textbf{Quotient.} \end{array}$$

The number 254 is the sum of the quotients of all of these parts by 237; it is, therefore, according to Article 38, the required quotient.

In like manner we may treat all similar cases; hence, the following

RULE.

I. Find how many times the divisor is contained in the fewest possible figures on the left of the dividend for the first figure of the quotient; multiply the divisor by this figure and subtract the product from the figures used.

II. Annex to the remainder thus found the next figure of the dividend, and find how many times the divisor is contained in the result for the second figure of the quotient; multiply the divisor by this figure and subtract as before.

III. Continue this operation till all the figures of the quotient have been found.

In applying the preceding rule, it is found convenient to write the *divisor* on the *left* and the *quotient* on the *right* of the dividend. Should there be a remainder after the operation is completed, it is to be treated as explained in short division.

The method of proof is the same as for short division. In what follows let every example be proved.

EXAMPLES.

Perform the following divisions:

(1.)

Dividend.
Divisor, 53)13012(245 Quotient.
106
241
212
292
265
27 Remainder.

PROOF.

245	Quotient.
53	Divisor.
735	Partial products
1225	
27	Remainder.
13012	Dividend.

The practical method of adding the remainder to the product of the quotient and the divisor is indicated in the above example.

(2.)

36)7452(207
72
252
252
0

(3.)

124)373116(3009
372
1116
1116
0

In example (2), the partial dividend, 25, is smaller than the divisor; we therefore write 0 for the second figure of the quotient and bring down 2, the next figure of the dividend; the quotient of 252 by 36 is 7.

In example (3), both 11 and 111 are smaller than the divisor; we therefore write two ciphers in the quotient, bring down 6, and proceed as before.

4. Divide 7,812 by 36. *Ans.* 217.
5. Divide 16,758 by 49. *Ans.* 342.
6. Divide 14,464 by 64. *Ans.* 226.

Perform the following indicated divisions:

7. 72,518 ÷ 718.
8. 40,698 ÷ 399.
9. 38,214 ÷ 386.
10. 51,171 ÷ 111.
11. 10,368 ÷ 144.
12. 27,264 ÷ 96.
13. 88,534 ÷ 184.
14. 20,615 ÷ 95.
15. 45,579 ÷ 209.
16. 51,867 ÷ 112.
17. 309,927 ÷ 309.
18. 765,870 ÷ 98.

The dividend may be either an abstract, or a denominate number; in the former case, both the divisor and the quotient are also abstract; in the latter case, one of them is similar to the dividend and the other is abstract.

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Divide 3,072 by 8.

20. Divide 2,304 *lbs.* by 12.
21. Divide 1,360 *yds.* by 16 *yds.*
22. Divide 48,708 *ft.* by 27 *ft.*
23. Divide $71,556 by 201.
24. Divide 30,056 *things* by 884 *things.*
25. Divide 68.541 *houses* by 341 *houses.*
26. Divide $14,874 by 402.
27. Divide 14,430 *lbs.* by 74 *lbs.*
28. Divide 300,360 *miles* by 120.
29. Divide 61,712 *horses* by 304.
30. Divide 722,631 by 91.
31. Divide 317,094 by 82.
32. Divide 1,731,195 by 73.
33. Divide 7,318,080 by 55.
34. Divide 76,131,702 by 46.
35. Divide 31,231,737 by 37.
36. Divide 13,261,467 by 381.
37. Divide 1,281,524 by 761.
38. Divide 13,189,212 by 937.
39. Divide 728,807 by 731.
40. Divide 762,294 by 754.
41. Divide 611,398 by 107.
42. Divide 16,865,628 by 438.
43. Divide 962,928 by 108.
44. Divide 7,281,711 by 999.

Division by Composite Numbers.

42. If the divisor is a composite number, we may divide by each of its factors in succession, (Art. **34**).

EXAMPLES.

1. Divide 19,866 by 77, that is, by 11 × 7.

OPERATION.

$$\begin{array}{r|l} 11 & 19866 \\ \hline 7 & 1806 \quad \text{First result.} \\ \hline & 258 \quad \text{Quotient.} \end{array}$$

PROOF.

$$\begin{array}{rl} 258 & \text{Quotient.} \\ 7 & \\ \hline 1806 & \text{First product.} \\ 11 & \\ \hline 19866 & \text{Dividend.} \end{array}$$

There may be a remainder after each operation. The method of finding the *true remainder* in such cases is shown in the following example:

2. Divide 1,592 by 35, that is, by 7 × 5.

OPERATION.

$$\begin{array}{r|l} 7 & 1592 \\ \hline 5 & 227 \;\ldots\; 3 \quad \text{First partial remainder.} \\ \hline & 45 \;\ldots\; 2 \quad \text{Second partial remainder.} \end{array}$$

The first of the above steps shows that 1,592 contains 227 *sevens* and 3 *units;* the second step shows that 227 *sevens* contains 45 *thirty-fives* and 2 *sevens;* but 2 *sevens* is equal to 14 units; hence, 1,592 contains 45 times 35, with a remainder equal to 14 + 3, or 17, that is, the quotient of 1,592 by 35 is $45\frac{17}{35}$. Hence, to find the true remainder in such cases, we multiply the *second* partial remainder by the *first* divisor, and add the *first* partial remainder to the result.

The same principle holds good when the divisor is composed of 3 or more factors.

3. Divide 7,445 by 84, or by 7 × 4 × 3.

Here we divide successively by 7, 4, and 3; the final quotient is 88, the first remainder is 4, the second is 3, and the third is 1. To find the true remainder we multiply the *third* remainder by the *second* divisor and add the *second* remainder; we then multiply this result by the *first* divisor and add the *first* remainder. Thus, $(1 \times 4 + 3) \times 7 + 4 = 7 \times 7 + 4 = 53$, *true remainder.*

4. Divide 8,514 by 99. *Ans.* 86.
5. Divide 15,336 by 72. *Ans.* 213.
6. Divide 93,312 by 108. *Ans.* 864.

To divide by 10, 100, 1,000, &c., we cut off as many figures from the right as there are ciphers in the divisor; the figures on the left form the quotient and those on the right are the remainder, (Art. 30).

7. Divide 8,759 by 100. *Ans.* $87\frac{59}{100}$.
8. Divide 3,874 by 1,000. *Ans.* $3\frac{874}{1000}$.
9. Divide 5,800 by 100. *Ans.* 58.
10. Divide 81,000 by 1,000. *Ans.* 81.

If the divisor is composed of a significant part followed by ciphers, we cut off the ciphers and also the same number of figures from the right of the dividend; we then divide the remaining part of the dividend by the significant part of the divisor; to find the true remainder we annex to the partial remainder the figures cut off from the dividend.

11. Divide 37,843 by 2,500.

OPERATION.

25,00) 378,43 (15

25

128

125

True remainder, 343

Explanation.—The operation is equivalent to dividing first by 100 and then by 25. The first partial remainder is 43, the second partial remainder is 3, and the first divisor is 100; hence, by the rule, we have the true remainder equal to $3 \times 100 + 43$, or 343.

12. Divide 98,742 by 1,700. *Ans.* $58\frac{142}{1700}$.
13. Divide 8,436 by 2,100. *Ans.* $4\frac{36}{2100}$.
14. Divide 8,566 by 2,500. *Ans.* $3\frac{1066}{2500}$.

If both dividend and divisor terminate in ciphers, we strike off from the right of each as many as are common to both, and then perform the division.

15. Divide 875,000 by 2,500.

Explanation.—Striking off two ciphers from each is equivalent to dividing both by 100, which obviously does not affect the resulting quotient.

OPERATION.

$$\begin{array}{r|l} 25{,}00 \;)\; & 8750{,}00 \;(\; 350 \\ & \underline{75} \\ & 125 \\ & \underline{125} \\ & \quad 0 \end{array}$$

16. Divide 1,831,200 by 240. *Ans.* 7,630.
17. Divide $1,350,500 by 3,650. *Ans.* $370.
18. Divide 687,500*yds.* by 27,500*yds.*
19. Divide 201,600 by 3,600.
20. Divide 41,580 by 540.
21. Divide 71,820 by 87.
22. Divide 1,749,600 by 360.
23. Divide 98,710 by 8,400.
24. Divide 66,920 by 8,800.

PRACTICAL PROBLEMS.

43. In applying the rules for division to practical problems, two cases may arise:

1°. It may be required to divide a collection of things into a number of equal parts; in this case the divisor is abstract and the quotient is similar to the dividend. Thus, 8*yds.* ÷ 4 = 2*yds.*

2°. It may be required to divide a number into parts, each equal to a given number; in this case the dividend and divisor are similar and the quotient is abstract. Thus, 8*yds.* ÷ 4*yds.* = 2.

In both cases, the division is performed as though the numbers were abstract, and then the unit of the quotient is determined by the nature of the question.

1. An estate worth $41,185 was divided equally among 5 persons; what was the share of each? *Ans.* $8,237.

2. An estate worth $41,185 was divided equally among a certain number of heirs so that each received $8,237; how many heirs were there? *Ans.* 5.

3. The capital of a joint-stock company is $13.125 and is divided into 175 shares; what is the value of each share? *Ans.* $75.

4. If a ship sails 5,712 miles in 48 days, how many miles does she sail per day? *Ans.* 119.

5. If a ship sails 114 miles in 1 day, how many days will it take her to sail 2,622 miles? *Ans.* 23.

6. A farmer paid $13,216 for a farm of 112 acres; how much did he pay per acre?

7. How many acres of land can be bought with $26,432, at the rate of $59 per acre?

8. In a field of corn there are 21,033 hills and each row contains 171 hills; how many rows are there?

9. The mean diameter of the earth is 7,912 miles, and that of the sun is 854,496 miles; how many diameters of the earth are there in the sun's diameter?

10. A grocer bought 55,664 pounds of flour put up in barrels, each of which contained 196 pounds; how many barrels were there in the lot?

11. There are 4,032 yards of cloth in 96 equal pieces; how many yards are there in each piece?

12. A field produces 3,404 bushels of oats at the rate of 37 bushels per acre; how many acres are there in the field?

13. Twenty pieces of cloth contain 39 yards each; 32 pieces contain 38 yards each; and 17 pieces contain 43 yards each; how many yards in all?

14. A merchant bought 175 yards of cloth at 7 dollars per yard and afterwards sold 72 yards at 9 dollars per yard and the remainder at 8 dollars per yard; how much did he gain?

15. A dealer bought 27 barrels of flour at $14 per barrel and gave in exchange 32 cords of wood at $8 per cord and paid the balance in cash; how much cash did he pay?

16. A man's income is $3,150 per year and his expenses are $2,817 per year; how much can he save in 6 years?

17. A farmer bought 32 acres of land at $95 per acre, 71 acres at $47 per acre, 38 acres at $62 per acre, and 19 acres at $88 per acre; what did he pay for the whole?

18. The factors of one number are 19, 17, and 23; of another number, 31, 29, and 11; and of a third number, 77 and 83; what is the sum of the numbers?

19. Two men start from the same point and travel in opposite directions; the first travels at the rate of 43 miles per day, and the second at the rate of 37 miles per day; how far apart are they at the end of 7 days?

20. A farmer bought 6 oxen at $65 each, 12 cows at $42 each, and 142 sheep at $6 each; what did he pay for the whole?

21. In a freight car there are 6 boxes of goods, each weighing 382 pounds; 13 barrels, each weighing 218 pounds; and 37 bags, each weighing 179 pounds; how many pounds in all?

22. In 19 bales of cloth, each bale containing 16 pieces, and each piece containing 42 yards, how many yards?

23. What number multiplied by 86 will give the same product as 163 multiplied by 430?

24. How many yards of muslin at 14 cents a yard must be given in exchange for 35 bushels of oats at 56 cents a bushel?

25. A., B., and C. enter into partnership; A. puts in $7,200, B. puts in $700 more than A., and C. puts in $550 less than A. and B. together; what is the capital of the firm?

26. A.'s income is 5 times B.'s, B.'s income is 3 times C.'s, and C.'s income is $1,325; what is the entire income of A., B., and C.?

27. A farmer bought 154 acres of land at $64 per acre, and sold the whole for $11,704; how many dollars did he gain per acre?

28. The distance from New York to Albany is 144 miles, and each mile contains 5,280 feet; how many hours of 60 minutes each will it take a man to walk from one city to the other, if he walks at the rate of 352 feet per minute?

29. The sum of two numbers is 10,370, and the second is 4 times the first; what are the numbers?

30. The first of three numbers is 24, the second is 3 times the first, and the third is 4 times the sum of the

first and second; what is the difference between the second and third? *Ans.* 312.

31. Write down 4,617, multiply it by 12, divide the product by 9, add 365 to the quotient, and from the sum subtract 5,521; what is the final result?

Ans. 1,000.

EXAMINATION QUESTIONS.

(35.) What is division? Define the dividend; the divisor; the quotient; and the remainder. When is division exact? What are factors? **(36.)** What does the sign of division indicate when placed between two numbers? In what other ways may division be indicated? **(37.)** What is the relation between multiplication and division? How may the multiplication table be employed as a division table? **(38.)** State the leading principle of division. **(39.)** What is short division? What is long division? **(40.)** Give the rule for short division. Method of Proof? **(41.)** Give the rule for long division. **(42.)** How do you divide by a composite number? What is the method of determining the true remainder? How do you divide by 10, 100, 1,000, etc.? How do you divide by a number that ends in ciphers? How do you divide when both dividend and divisor terminate in ciphers? **(43.)** Explain the two cases in practical problems. How is the unit of the answer determined?

V. FACTORING AND CANCELLING.

Definitions.

44. A *factor* of a number is one of its exact divisors, (Art. **35**). Thus, 2, 3, and 4, are factors of 12.

Neither the number itself nor the number 1 is regarded as a factor.

45. A **composite** number is a number composed of two or more integral factors, (Art. **34**). Thus, 15 is a composite number, because it is the product of 3 and 5.

A **prime** number is one that cannot be separated into integral factors. Thus, 2, 3, 5, etc., are prime numbers.

46. **Factoring** is the operation of separating a number into integral factors.

The factors of a number may be either prime, or composite. Composite factors may themselves be factored, and so on, till all the factors are prime. Thus, $24 = 2 \times 12 = 2 \times 2 \times 6 = 2 \times 2 \times 2 \times 3$; hence, the prime factors of 24 are 2, 2, 2, and 3.

Principles of Factoring.

47. The operation of resolving a number into prime factors depends on the following principles:

1°. *A number is equal to the continued product of all its prime factors.*

2°. *If a number is divided by one of its prime factors, the quotient is equal to the continued product of all the others.*

Operation of Factoring.

48. Let it be required to resolve 210 into its prime factors.

Explanation.—We first divide by 2, which is a prime factor; we next divide the first quotient by 3, which is also a prime factor; we then divide the second quotient by 5, and find 7 for a quotient. The numbers 2, 3, 5, and 7 are the required factors, that is,

OPERATION.

$$\begin{array}{r} 2\overline{)210} \\ 3\overline{)105} \\ 5\overline{)35} \\ 7 \end{array}$$

$$210 = 2 \times 3 \times 4 \times 5.$$

In like manner other composite numbers may be factored; hence, the following

RULE.

Divide the given number by one of its prime factors; then divide the quotient by one of its prime factors; and so on, till a quotient is found that is a prime number; the several divisors and the last quotient are the required factors.

It will be found convenient to begin the division with the smallest prime factor.

EXAMPLES.

Resolve the following numbers into their prime factors:

1. 42. *Ans.* 2 × 3 × 7.
2. 180. *Ans.* 2 × 2 × 3 × 3 × 5.
3. 378. *Ans.* 2 × 3 × 3 × 3 × 7.
4. 330. *Ans.* 2 × 3 × 5 × 11.

If there are more than two factors in any indicated product, the sign of multiplication may be replaced by a simple dot; thus, 3.4.5 is equivalent to 3 × 4 × 5.

5. 770. *Ans.* 2.5.7.11.
6. 1,575. *Ans.* 3.3.5.5.7.
7. 3,850. *Ans.* 2.5.5.7.11.
8. 1,225. *Ans.* 5.5.7.7.
9. 2,310. *Ans.* 2.3.5.7.11.
10. 122,850. *Ans.* 2.3.3.3.5.5.7.13.

The operation of factoring is performed principally by inspection and trial. It may sometimes be facilitated by using the following

Table of Prime Numbers from 1 to 150.

1	2	3	5	7	11	13	17	19
23	29	31	37	41	43	47	53	59
61	67	71	73	79	83	89	97	101
103	107	109	113	127	131	137	139	149

11. 402. *Ans.* 2 . 3 . 67.

12. 1,659. *Ans.* 3 . 7 . 79.

13. 3,290. *Ans.* 2 . 5 . 7 . 47.

14. 1,554. *Ans.* 2 . 3 . 7 . 37.

15. 1,095. *Ans.* 3 . 5 . 73.

If the final digit of a number is 0, 2, 4, 6, or 8, the number is divisible by 2.

If the sum of the digits of a number is divisible by 3, the number itself is divisible by 3.

If the final digit of a number is 5, the number is divisible by 5.

16. 930. *Ans.* 2 . 3 . 5 . 31.

17. 1,455. *Ans.* 3 . 5 . 97.

18. 3,685. *Ans.* 5 . 11 . 67.

19. 1,738. *Ans.* 2 . 11 . 79.

20. 651. *Ans.* 3 . 7 . 31.

Cancellation.

49. **Cancellation** is the operation of striking out one or more factors from an indicated product.

The operation is performed by drawing a line across each factor that is to be struck out, or *cancelled*. Thus, the expression $\cancel{3} . \cancel{5} . 13$ indicates that the factors 3 and 5 have been cancelled.

Object and Principles of Cancellation.

50. The operation of division may often be shortened by cancelling factors common to both dividend and divisor.

This method of shortening the operation of division depends on the following principles:

1°. *Striking out a factor of a number is equivalent to dividing the number by that factor.*

2°. *If both dividend and divisor are divided by the same number the quotient is not changed.*

APPLICATIONS.

51. Let it be required to divide 210 by 182.

OPERATION.

$$\frac{210}{182}=\frac{\cancel{2}.3.5.\cancel{7}}{\cancel{2}.\cancel{7}.13}=\frac{15}{13}=1\tfrac{2}{13}.$$

Explanation.—Having indicated the division, we resolve both dividend and divisor into prime factors; we next cancel all that are common to both. We then divide the product of all the factors that remain in the dividend by the product of all that remain in the divisor.

In like manner we may treat all similar cases; hence, the following

RULE.

I. Resolve both dividend and divisor into prime factors and strike out all that are common to both.

II. Divide the continued product of the factors that remain in the dividend by the continued product of those that remain in the divisor.

EXAMPLES.

Perform the following indicated operations by the method of cancellation:

1. $2{,}310 \div 570$. *Ans.* $\frac{77}{19} = 4\frac{1}{19}$.

2. $122{,}850 \div 10{,}800$. *Ans.* $\frac{91}{8} = 11\frac{3}{8}$.

3. $\frac{96 \times 48 \times 11}{36 \times 72 \times 4}$. *Ans.* $\frac{44}{9} = 4\frac{8}{9}$.

If all the factors in either dividend or divisor are struck out, the unit 1 is left. If it is in the dividend it must be retained; if it is in the divisor it may be omitted.

4. $\frac{70}{1120} = \frac{\cancel{2} \cdot \cancel{5} \cdot \cancel{7}}{\cancel{2} \cdot 2 \cdot \cancel{5} \cdot \cancel{7} \cdot 8}$. *Ans.* $\frac{1}{16}$.

5. $\frac{6825}{105} = \frac{\cancel{3} \cdot \cancel{5} \cdot 5 \cdot \cancel{7} \cdot 13}{\cancel{3} \cdot \cancel{5} \cdot \cancel{7}}$. *Ans.* 65.

6. $\frac{3087}{58653} = \frac{\cancel{3} \cdot \cancel{3} \cdot \cancel{7} \cdot \cancel{7} \cdot \cancel{7}}{\cancel{3} \cdot \cancel{3} \cdot \cancel{7} \cdot \cancel{7} \cdot \cancel{7} \cdot 19}$. *Ans.* $\frac{1}{19}$.

7. $\frac{24087}{21} = \frac{3 \cdot 7 \cdot 31 \cdot 37}{3 \cdot 7}$. *Ans.* 1147.

It is not necessary that the factors struck out should be prime, provided they are the same in both dividend and divisor.

8. Divide 144×56 by 96.

OPERATION.

$$\frac{144 \times 56}{96} = \frac{\cancel{12} \cdot 12 \cdot \cancel{8} \cdot 7}{\cancel{8} \cdot \cancel{12}} = 84.$$

9. Divide 168×216 by 42×54. *Ans.* 16.

If the product of two or more factors of the dividend is equal to the product of two or more factors of the divisor, all may be cancelled.

10. Divide $9 \times 24 \times 31$ by 72.

OPERATION.

$$\frac{3 \cdot \cancel{3} \cdot \cancel{24} \cdot 31}{\cancel{72}} = 93. \quad \textit{Ans.}$$

11. Divide $7 \cdot 42 \cdot 54$ by 294. *Ans.* 54.
12. Divide 66×154 by 121. *Ans.* 84.

If two factors, one in the dividend and one in the divisor, are divisible by the same number, they may be struck out and replaced by the resulting quotients.

13. Divide 36×7 by 27×11.

$$\textit{Ans.}\ \frac{36 \times 7}{27 \times 11} = \frac{4 \times 7}{3 \times 11} = \frac{28}{33}.$$

14. Divide $3 \cdot 26 \cdot 14$ by 42.
15. Divide 48×125 by 15.
16. Divide 342×6 by 36.
17. Divide $1,323 \times 5$ by 441.
18. Divide $3 \cdot 200 \cdot 9$ by 18×56.
19. Divide 2,910 by 1,110.
20. Divide 2,415 by 483.
21. Divide 3,115 by 595.

PRACTICAL PROBLEMS.

1. How many boxes of tea, each containing 24 pounds, at 75 cents a pound, must be given for 145 bags of wheat, each bag containing 2 bushels, at 180 cents a bushel? *Ans.* 29 *boxes.*

2. A. worked 18 days at $3 per day, for which he received 6 barrels of flour; how much was the flour worth per barrel? *Ans.* $9.

3. A man buys 3 pieces of cotton cloth, each containing 42 yards, at 13 cents per yard, and pays for it in butter at 21 cents per pound; how many pounds of butter must he give? *Ans.* 78 *lbs.*

4. Bought 15 barrels of apples, each containing 3 bushels, at 84 cents a bushel; how many cheeses, each weighing 45 pounds, at 12 cents per pound, will pay for the apples? *Ans.* 7 *cheeses.*

EXAMINATION QUESTIONS.

(44.) What is a factor of a number? Is a number a factor of itself? Is 1 a factor of any number? **(45.)** What is a composite number? A prime number? Illustrate. **(46.)** What is factoring? **(47.)** What are the principles of factoring? **(48.)** What is the rule for resolving a number into prime factors? When there are more than two factors in a product, how may the multiplication be indicated? **(49.)** What is cancellation? How is it indicated? **(50.)** What is the use of cancellation? On what principle does its use depend? **(51.)** Give the rule for shortening division by cancellation.

VI. GREATEST COMMON DIVISOR, AND LEAST COMMON MULTIPLE.

Definitions.

52. A **Common Divisor** of two or more numbers is a number that will exactly divide them all. Thus, 4 is a common divisor of 8, 16, and 32.

The **Greatest Common Divisor** of two or more numbers is the greatest number that will exactly divide them all. Thus, 8 is the greatest common divisor of 8, 16, and 32.

Numbers that have no common divisor, except 1, are said to be *prime with respect to each other*.

Methods and Principles.

53. There are two methods of finding the greatest common divisor of two or more numbers: 1°. *By factors;* and 2°. *By continued division.*

Both methods depend on the following principles:

1°. *Any factor common to two or more numbers is a common divisor of those numbers.*

2°. *The greatest common divisor is equal to the continued product of the prime factors that are common to all the numbers.*

Method by Factors.

54. Let it be required to find the greatest common divisor of 126, 210, and 546.

Resolving the given numbers into prime factors, we have,

$126 = 2 . 3 . 3 . 7$, $210 = 2 . 3 . 5 . 7$, and $546 = 2 . 3 . 7 . 13$.

The factors 2, 3, and 7 are common to all the given numbers, and they are the only factors that are common; hence, their product is the greatest common divisor of the given numbers; denoting the greatest common divisor by the initials *g. c. d.*, we have *g. c. d.* = 42.

All similar cases may be treated in like manner; hence, the following

RULE.

I. Resolve the numbers into prime factors.

II. Find the continued product of the prime factors common to all the numbers.

EXAMPLES.

1. Find the greatest common divisor of 168, 216, and 408. *Ans.* 24.

2. What is the *g. c. d.* of 408 and 740? *Ans.* 4.

3. What is the *g. c. d.* of 90, 315, and 810? *Ans.* 45.

4. What is the *g. c. d.* of 441 and 567? *Ans.* 63.

5. Find the *g. c. d.* of 195, 285, and 315. *Ans.* 15.

6. Find the *g. c. d.* of 462, 726, and 1,254.

7. Find the *g. c. d.* of 1,470, 2,310, and 2,730.

8. Find the *g. c. d.* of 320, 1,216, and 6,400.

9. Find the *g. c. d.* of 540, 648, and 756.

10. What is the *g. c. d.* of 567, 648, and 729

Additional Principle.

55. The greatest common divisor of two numbers will divide their remainder after division.

For, let 8 be the greatest common divisor of 88 and 24, and also let it be taken as a *unit:* the two numbers, expressed in terms of this unit, are 11 *eights* and 3 *eights*. Dividing the greater by the less, we have,

Divisor. **Dividend.**
3 *eights*)11 *eights*(3 **Quotient.**
9 *eights.*
2 *eights.* **Remainder after division.**

Here we see that 8, which is the greatest common divisor of the given numbers, is also a divisor of their remainder after division.

Any number that will divide 3 *eights* will divide 3 times 3 *eights*, or 9 *eights*; consequently, any number that will divide the *smallest number* and the *remainder after division* will also divide the *larger number*; hence, the following *principle:*

The greatest common divisor of two numbers is the same as the greatest common divisor of the smaller number and of their remainder after division.

Method by Continued Division.

56. Let it be required to find the greatest common divisor of 88 and 24.

Explanation.—Dividing 88 by 24, we find 16 for a remainder; then dividing 24 by 16, we find 8 for a remainder; then dividing 16 by 8, we find 0 for a remainder. Hence, 8 is the greatest common divisor of 16 and 24, (Art. 55); it is therefore the greatest common divisor of 24 and 88.

In like manner, all similar cases may be treated; hence, the following

OPERATION.

```
24)88(3
   72
   16)24(1
      16
       8)16(2
         16
          0
```

RULE.

I. Divide the greater number by the less and find the remainder.

II. Take the divisor for a new dividend and the remainder for a divisor, and proceed as before.

III. Continue the operation till a remainder is found that will exactly divide the preceding divisor; this will be the greatest common divisor of the given numbers.

EXAMPLES.

1. Find the *g. c. d.* of 3,471 and 1,869. *Ans.* 267.

2. What is the *g. c. d.* of 1,584 and 2,772?
Ans. 396.

3. What is the *g. c. d.* of 2,898 and 7,866?
Ans. 414.

4. What is the *g. c. d.* of 3,724 and 5,852?
Ans. 532.

5. Find the *g. c. d.* of 3,444 and 2,268.

6. What is the *g. c. d.* of 10,395 and 16,797?

7. What is the *g. c. d.* of 667 and 391?

8. What is the *g. c. d.* of 10,353 and 14,877?

To find the greatest common divisor of more than two numbers, begin with the least and find the greatest common divisor of two, then of that result and the third, then of that and the fourth, and so on to the last.

9. Find the *g. c. d.* of 805, 1,311, and 1,978.
Ans. 23.

10. What is the *g. c. d.* of 504, 5,292, and 1,512?
Ans. 252.

11. What is the *g. c. d.* of 837, 1,134, and 1,347?
Ans. 3.

12. Find the *g. c. d.* of 492, 744, and 1,044.
Ans. 12.

13. Find the *g. c. d.* of 944, 1,488, and 2,088.

14. Find the *g. c. d.* of 72, 100, 168, and 104.

15. Find the *g. c. d.* of 756, 864, 972, and 1,080.

16. What is the *g. c. d.* of 1,554, 1,998, 2,220, and 2,886?

LEAST COMMON MULTIPLE.

Definitions.

57. A **Multiple** of a number is a number that is exactly divisible by it. Thus, 12 is a multiple of 6.

A **Common Multiple** of two or more numbers is a number that is exactly divisible by each. Thus, 48 is a common multiple of 4, 6, and 8.

The **Least Common Multiple** of two or more numbers is the least number that is exactly divisible by each. Thus, 24 is the least common multiple of 4, 6, and 8.

Principles.

58. The operation of finding the least common multiple of two or more numbers depends on the following *principles:*

1°. *The least common multiple must contain every prime factor of each number.*

2°. *It must contain every prime factor the greatest number of times it enters any of the numbers.*

Operation of Finding the Least Common Multiple.

59. Let it be required to find the least common multiple of 12, 25, and 90.

Resolving the given numbers into prime factors, we have,

$$12 = 2 . 2 . 3, \quad 25 = 5 . 5, \quad \text{and} \quad 90 = 2 . 3 . 3 . 5.$$

The least common multiple must contain the factor 2 twice in order to be divisible by 12, it must contain the factor 3 twice in order to be divisible by 90, and it must contain the factor 5 twice in order to be divisible by 25. Hence, if we denote the least common multiple by the initials *l. c. m.*, we have,

$$l.\ c.\ m. = 2 . 2 . 3 . 3 . 5 . 5 = 900.$$

The method of factoring may often be simplified as follows:

OPERATION.

$$\begin{array}{r|rrr} 5 & 12, & 25, & 90 \\ \hline 3 & 12, & 5, & 18 \\ \hline 2 & 4, & 5, & 6 \\ \hline & 2, & 5, & 3 \end{array}$$

$$l.\ c. m. = 2 . 2 . 3 . 3 . 5 . 5 = 900$$

Explanation.—Having written the numbers in a line, we see by inspection that 5 is a prime factor of 25 and 90. We therefore write it on the left as a divisor. Dividing 25 and 90 by it, we write the quotients and also the undivided number 12 in the second line. We then see that 3 is a prime factor of 12 and 18; writing it on the left, and proceeding as before, we find the third line of numbers. We then see that 2 is a prime factor of 4 and 6; writing it on the left and proceeding as before, we find the fourth line of numbers, which are prime with respect to each other. Here we have resolved 12 into the factors 3, 2, and 2; 25 into the factors 5 and 5; and 90 into the factors 5, 3, 2, and 3; hence, from the principles laid down, the least common multiple is equal to the continued product of 2 × 2, 3 × 3, and 5 × 5, or to 900.

In like manner we can find the least common multiple of other groups of numbers; hence the following

RULE.

I. Write the numbers in a line and divide by any prime factor that is contained in two or more, writing the quotients and the undivided numbers in the line below.

II. Then operate on the second line of numbers in the same manner, and so on, till a line of numbers is found that are prime with respect to each other.

III. Find the continued product of the numbers in the last line and of the divisors used; this will be the least common multiple of the given numbers.

EXAMPLES.

Find the least common multiple of the following groups of numbers:

1. 3, 4, 8, and 12. *Ans.* 24.
2. 6, 7, 8, 9, and 10. *Ans.* 2,520.
3. 4, 6, 9, 14, and 16. *Ans.* 1,008.
4. 12, 48, 18, and 70. *Ans.* 5,040.
5. 14, 20, 198, and 210. *Ans.* 13,860.
6. 8, 18, 20, and 70. *Ans.* 2,520.
7. 9, 18, 27, 36, 54, and 45.
8. 7, 15, 21, 28, and 35.
9. 15, 16, 18, 20, and 24.
10. 49, 14, 84, 168, and 98.
11. 84, 100, and 224.
12. 49, 56, 63, and 84.

EXAMINATION QUESTIONS.

(52.) What is a common divisor of two or more numbers? What is the greatest common divisor of two or more numbers? When are numbers prime with respect to each other? (53.) What general principles are used in finding the greatest common divisor? (54.) Give the rule for finding the greatest common divisor by the method of factors. (55.) What additional principle is used? (56.) Give the rule for finding the greatest common divisor by the method of continued division. How do you find the greatest common divisor of more than two numbers? (57.) What is a multiple of a number? What is a common multiple of two or more numbers? What is their least common multiple? (58.) Give the principles used in finding the least common multiple. (59.) Give the rule for finding the least common multiple.

CHAPTER III.

FRACTIONS.

I. COMMON FRACTIONS.

Definitions.

60. If an integral unit is divided into equal parts, each part is called a *fractional unit.*

If the integral unit is divided into two equal parts, each is called *a half*; if into three, each is called *a third*; if into four, each is called *a fourth*; and so on.

Fractional units may be written and read as shown below:

$$\frac{1}{2},\quad \frac{1}{3},\quad \frac{1}{4},\quad \frac{1}{5},\quad \frac{1}{6},\quad \frac{1}{7},\quad \text{etc.}$$

one-half, one-third, one-fourth, one-fifth, one-sixth, one-seventh, etc.

The Reciprocal of a Number is 1 divided by that number. Thus, $\frac{1}{2}$ is the reciprocal of 2, $\frac{1}{3}$ the reciprocal of 3, and so on.

61. **A Fraction** is a fractional unit, or a collection of fractional units; thus, *one-half*, *two-thirds*, *four-ninths*, etc. are fractions.

Fractions may be written and read as shown below:

$$\frac{3}{4},\quad \frac{2}{9},\quad \frac{5}{7},\quad \frac{8}{11},\quad \text{etc.}$$

three-fourths, two-ninths, five-sevenths, eight-elevenths, etc.

Fractions written in this manner are called *vulgar*, or *common fractions.*

Common fractions are expressed by two numbers, one written above the other, with a line between them. The number below the line is called the *denominator*, and the one above it is called the *numerator*. Thus, in the fraction $\frac{3}{4}$, 4 is the denominator and 3 is the numerator. Both numerator and denominator are called *terms* of the fraction.

The denominator indicates the number of equal parts into which the unit 1 is divided, and the numerator shows how many of these parts are taken. Thus, in the fraction $\frac{3}{4}$, the denominator indicates that 1 is divided into 4 equal parts, and the numerator shows that 3 of these are taken.

62. A fraction is equal to its numerator divided by its denominator. Thus, $\frac{3}{4}$ is equal to $3 \div 4$; for, if each of the units in 3 is divided into four equal parts, we shall have 12 such parts, each equal to $\frac{1}{4}$, that is, we shall have 12 *fourths;* but 12 *fourths* divided by 4 is equal to 3 *fourths*, or to $\frac{3}{4}$, that is, $3 \div 4$ is equal to $\frac{3}{4}$.

63. From Articles **61** and **62** we see that a fraction may be regarded either as a number, or as an indicated division:

1°. When regarded as a number, the unit is *fractional* and equal to the reciprocal of the denominator. Thus, $\frac{3}{4}$ is a collection of 3 units, each equal to $\frac{1}{4}$, that is, $\frac{3}{4} = 3 \times \frac{1}{4}$.

2°. When regarded as an indicated division, the numerator is the dividend and the denominator is the divisor. Thus, $\frac{3}{4} = 3 \div 4$.

Kinds of Fractions.

64. A **Proper Fraction** is one in which the numerator is less than the denominator; as, $\frac{3}{4}$, $\frac{5}{6}$.

An **Improper Fraction** is one in which the numerator is equal to, or greater than the denominator; as, $\frac{4}{4}$, $\frac{8}{3}$.

If the numerator is equal to the denominator, the fraction is equal to 1; a proper fraction is less than 1; an improper fraction is equal to, or greater than 1.

A **Mixed Number** is a number composed of an integral and a fractional part; as, $2\frac{1}{2}$, $3\frac{1}{7}$.

A **Simple Fraction** is one in which both terms are whole numbers; as, $\frac{3}{7}$, $\frac{5}{3}$.

A **Complex Fraction** is one that has at least one of its terms fractional; as, $\frac{\frac{1}{7}}{5}$, $\frac{3}{14\frac{1}{2}}$, $\frac{15}{(\frac{1}{4})}$, $\frac{7\frac{1}{2}}{8\frac{1}{4}}$.

A **Compound Fraction** is a fractional part of a fraction, or mixed number; as, $\frac{1}{2}$ of $\frac{3}{4}$, $\frac{1}{3}$ of $2\frac{1}{4}$.

The parts of which a compound fraction is composed are called *partial fractions*. Thus, $\frac{1}{2}$ and $\frac{3}{5}$ are the partial fractions of $\frac{1}{2}$ of $\frac{3}{5}$.

A whole number may be regarded as a fraction whose denominator is 1. Thus, $8 = \frac{8}{1}$.

Fundamental Principles.

65. The numerator shows how many times the fractional unit is taken; hence, the following principles:

1°. *Multiplying the numerator of a fraction by any number is equivalent to multiplying the fraction by that number.*

2°. *Dividing the numerator of a fraction by any number is equivalent to dividing the fraction by that number.*

The denominator shows the number of equal parts into which we divide the unit 1 to obtain the fractional unit; hence, the following principles:

3°. *Multiplying the denominator of a fraction by any number is equivalent to dividing the fraction by that number.*

4°. *Dividing the denominator of a fraction by any number is equivalent to multiplying the fraction by that number.*

If we compare principles 1° and 3°, and also principles 2° and 4°, we have the following:

5°. *Multiplying both terms of a fraction by the same number does not alter its value.*

6°. *Dividing both terms of a fraction by the same number does not alter its value.*

REDUCTION OF FRACTIONS.

66. Reduction is the operation of changing the form of a number without altering its value.

The following methods of reducing fractions depend on the principles just deduced.

1°. *To reduce a whole number to the form of a fraction having a given fractional unit.*

67. Let it be required to reduce 8 to the form of a fraction having the fractional unit $\frac{1}{5}$:

Explanation.—Having written 8 under the form of a fraction, $\frac{8}{1}$, (Art. **64**), we multiply both of its terms by 5, in accordance with principle 1°; the result, $\frac{40}{5}$, is of the required form.

OPERATION.

$$\frac{8}{1} = \frac{8 \times 5}{1 \times 5} = \frac{40}{5}$$

In like manner we may treat all similar cases; hence, the following

RULE.

Multiply the number by the denominator of the given unit and write the product over that denominator.

EXAMPLES.

Perform the following reductions:

1. Reduce 12 to the unit $\frac{1}{4}$. *Ans.* $\frac{48}{4}$.
2. Reduce 14 to the unit $\frac{1}{5}$. *Ans.* $\frac{70}{5}$.
3. Reduce 7 to the unit $\frac{1}{3}$. *Ans.* $\frac{21}{3}$.
4. Reduce 19 to the unit $\frac{1}{5}$. *Ans.* $\frac{95}{5}$.

2. *To reduce a mixed number to a simple fraction.*

68. Let it be required to reduce $12\frac{3}{7}$ to the form of a simple fraction:

Explanation.—The mixed number $12\frac{3}{7}$ is equal to $12 + \frac{3}{7}$; reducing its integral part to the unit $\frac{1}{7}$, it becomes $\frac{84}{7} + \frac{3}{7}$, that is, the given number is equal to $\frac{1}{7}$ taken 84 + 3 times, or to $\frac{87}{7}$.

OPERATION.

$$12 + \frac{3}{7} = \frac{84}{7} + \frac{3}{7} = \frac{87}{7}$$

In like manner we may treat all similar cases; hence, the following

RULE.

Multiply the entire part by the denominator of the fraction and to the product add the numerator; then place the sum over the given denominator.

EXAMPLES.

Reduce the following mixed numbers to simple fractions:

1. $2\frac{1}{3}$. *Ans.* $\frac{7}{3}$.
2. $5\frac{3}{7}$. *Ans.* $\frac{38}{7}$.
3. $4\frac{5}{9}$. *Ans.* $\frac{41}{9}$.
4. $7\frac{2}{5}$. *Ans.* $\frac{37}{5}$.
5. $6\frac{3}{11}$.
6. $7\frac{3}{13}$.
7. $4\frac{11}{13}$.
8. $5\frac{2}{17}$.
9. $3\frac{2}{23}$.
10. $9\frac{7}{9}$.
11. $10\frac{4}{7}$.
12. $12\frac{3}{17}$.
13. $15\frac{2}{23}$.
14. $6\frac{12}{17}$.
15. $101\frac{3}{7}$.
16. $64\frac{2}{5}$.
17. $15\frac{7}{10}$.
18. $12\frac{5}{12}$.
19. $27\frac{2}{13}$.
20. $15\frac{3}{17}$.
21. $44\frac{9}{59}$.
22. $31\frac{9}{25}$.
23. $22\frac{7}{16}$.
24. $100\frac{7}{10}$.
25. $102\frac{9}{10}$.
26. $25\frac{3}{11}$.
27. $31\frac{2}{7}$.
28. $118\frac{11}{13}$.

3°. *To reduce an improper fraction to a mixed number.*

69. Let it be required to reduce $\frac{104}{22}$ to a mixed number.

Explanation.—We divide the numerator by the denominator and find the quotient, $4\frac{16}{22}$, which is of the required form.

In like manner we may treat all similar cases; hence, the following

OPERATION.

$22)104(4\frac{16}{22}$
88
16 Rem.

RULE.

Divide the numerator by the denominator; the quotient will be the required number.

If the remainder is 0, the division is exact and the given fraction is a whole number under a fractional form.

EXAMPLES.

Reduce the following improper fractions to mixed numbers:

1. $\frac{76}{6}$. *Ans.* $12\frac{4}{6}$.
2. $\frac{113}{9}$. *Ans.* $12\frac{5}{9}$.
3. $\frac{443}{12}$. *Ans.* $36\frac{11}{12}$.
4. $\frac{183}{55}$. *Ans.* $3\frac{18}{55}$.
5. $\frac{751}{24}$.
6. $\frac{580}{63}$.
7. $\frac{876}{157}$.
8. $\frac{640}{108}$.
9. $\frac{1000}{111}$.
10. $\frac{3175}{158}$.
11. $\frac{100}{27}$.
12. $\frac{775}{31}$.
13. $\frac{513}{133}$.
14. $\frac{256}{64}$.
15. $\frac{664}{29}$.
16. $\frac{455}{76}$.
17. $\frac{1261}{356}$.
18. $\frac{2565}{694}$.

4°. *To reduce a fraction to its lowest terms.*

70. A fraction is said to be in its *lowest terms* when its terms are prime with respect to each other, that is, when they have no common factor.

Let it be required to reduce the fraction $\frac{30}{105}$ to its lowest terms.

OPERATION.

$$\frac{30}{105} = \frac{2 \cdot 3 \cdot 5}{3 \cdot 5 \cdot 7} = \frac{2}{7}$$

Explanation.—We first resolve the terms of the fraction into prime factors, and then strike out those that are common to both, (Principle 6°, Art. 65); the resulting fraction is equivalent to the given one, and is in its lowest terms.

Since all similar cases may be treated in like manner, we have the following

RULE.

Resolve the terms of the fraction into prime factors, and cancel all that are common to both.

EXAMPLES.

Reduce the following fractions to their lowest terms:

1. $\frac{54}{81}$. *Ans.* $\frac{2}{3}$.
2. $\frac{63}{135}$. *Ans.* $\frac{7}{15}$.
3. $\frac{246}{342}$. *Ans.* $\frac{41}{57}$.

If the terms cannot be factored by inspection, find their greatest common divisor and divide them both by it.

4. $\frac{6465}{7335}$. *Ans.* $\frac{431}{489}$.

5. $\frac{105}{225}$.
6. $\frac{29}{87}$.
7. $\frac{451}{627}$.
8. $\frac{91}{119}$.
9. $\frac{400}{625}$.
10. $\frac{115}{253}$.
11. $\frac{121}{396}$.
12. $\frac{245}{364}$.
13. $\frac{189}{357}$.
14. $\frac{78}{198}$.
15. $\frac{55}{792}$.
16. $\frac{72}{1872}$
17. $\frac{221}{403}$.
18. $\frac{155}{217}$.
19. $\frac{96}{144}$.
20. $\frac{504}{616}$.
21. $\frac{805}{875}$.
22. $\frac{1872}{2016}$.
23. $\frac{3069}{3168}$.
24. $\frac{363}{605}$.
25. $\frac{1089}{1573}$.
26. $\frac{350}{1015}$.
27. $\frac{1309}{1785}$.
28. $\frac{504}{1848}$.
29. $\frac{5184}{6912}$.
30. $\frac{3168}{5148}$.

5°. *To reduce a compound fraction to a simple fraction.*

71. Let it be required to reduce $\frac{3}{5}$ of $\frac{7}{8}$ to a simple fraction.

Explanation. — The form of the given fraction indicates that $\frac{7}{8}$ is to be divided by 5 for a new unit, and that 3 of these units are to be taken; to divide $\frac{7}{8}$ by 5, we multiply its denominator by 5; to take that result 3 times, we multiply the numerator by 3; the result, $\frac{21}{40}$, is therefore equivalent to the given fraction.

OPERATION.

$$\frac{3}{5} \text{ of } \frac{7}{8} = \frac{3 \times 7}{5 \times 8} = \frac{21}{40}$$

Since all similar cases may be treated in the same manner, we have the following

RULE.

Find the continued product of the numerators of the partial fractions for a new numerator, and the continued product of their denominators for a new denominator.

EXAMPLES.

Reduce the following compound fractions to simple fractions:

1. $\frac{2}{3}$ of $\frac{5}{7}$.	*Ans.* $\frac{10}{21}$.
2. $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{7}{11}$.	*Ans.* $\frac{21}{110}$.
3. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{7}{13}$.	*Ans.* $\frac{56}{195}$.
4. $\frac{3}{7}$ of $\frac{4}{11}$ of $\frac{5}{13}$.	*Ans.* $\frac{60}{1001}$.

If there are any mixed or entire numbers entering into the compound fraction, let them be reduced to the form of simple fractions.

If there are any factors common to any of the numerators and any of the denominators, strike them out.

5. $\frac{3}{5}$ of $\frac{5}{7}$ of $1\frac{3}{4} = \frac{3}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{7}} \times \frac{\cancel{7}}{4} = \frac{3}{4}$. *Ans.*

6. $\frac{2}{3}$ of $\frac{3}{11}$ of $17 = \frac{2}{\cancel{3}} \times \frac{\cancel{3}}{11} \times \frac{17}{1} = \frac{34}{11}$. *Ans.*

7. $\frac{2}{3}$ of $\frac{3}{4}$ of 2.
8. $\frac{6}{7}$ of $9\frac{1}{2}$.
9. $\frac{2}{3}$ of $\frac{1}{2}$ of $7\frac{1}{4}$.
10. $\frac{5}{11}$ of $\frac{3}{5}$ of 4.
11. $\frac{7}{9}$ of $\frac{4}{5}$ of $2\frac{1}{5}$.
12. $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$.

6°. *To reduce fractions to a common denominator.*

72. Let it be required to reduce $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ to equivalent fractions having a common denominator.

Explanation.—We first find the least common multiple of all the denominators, which is 12; we then divide this by each denominator separately and multiply both terms of the corresponding fraction by the quotient. Thus, we divide 12 by 3 and then multiply both terms of $\frac{2}{3}$ by the quotient, which is 4, and so on.

In like manner we may treat all similar cases; hence, the following

OPERATION.

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12},$$

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12},$$

$$\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}.$$

RULE.

Find the least common multiple of all the denominators for a common denominator; divide this by each denominator separately and multiply the corresponding numerator by the quotient.

EXAMPLES.

Reduce each of the following groups of fractions to a common denominator:

1. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{7}{8}$, and $\frac{5}{12}$. *Ans.* $\frac{12}{24}$, $\frac{16}{24}$, $\frac{21}{24}$, and $\frac{10}{24}$.
2. $\frac{5}{8}$, $\frac{2}{5}$, $\frac{3}{4}$, and $\frac{4}{15}$. *Ans.* $\frac{75}{120}$, $\frac{48}{120}$, $\frac{90}{120}$, and $\frac{32}{120}$.
3. $\frac{1}{3}$, $\frac{3}{10}$, $\frac{5}{6}$, and $\frac{7}{12}$. *Ans.* $\frac{20}{60}$, $\frac{18}{60}$, $\frac{50}{60}$, and $\frac{35}{60}$.
4. $\frac{3}{7}$, $\frac{5}{14}$, and $\frac{11}{28}$. *Ans.* $\frac{12}{28}$, $\frac{10}{28}$, and $\frac{11}{28}$.

5. $\frac{3}{5}$, $\frac{35}{80}$, and $\frac{14}{200}$. *Ans.* $\frac{240}{400}$, $\frac{175}{400}$, and $\frac{28}{400}$.
6. $\frac{3}{7}$, $\frac{8}{14}$, $\frac{6}{28}$, and $\frac{6}{42}$. *Ans.* $\frac{36}{84}$, $\frac{48}{84}$, $\frac{18}{84}$, and $\frac{12}{84}$.

If the given fractions are in their lowest terms, the preceding rule will reduce them to their *least* common denominator; hence, we should reduce each fraction to its lowest terms before applying it.

Reducing the fractions in example 6, we have:

7. $\frac{3}{7}$, $\frac{4}{7}$, $\frac{3}{14}$, and $\frac{1}{7}$. *Ans.* $\frac{6}{14}$, $\frac{8}{14}$, $\frac{3}{14}$, and $\frac{2}{14}$.

Reduce each of the following groups to their least common denominator:

8. $\frac{2}{7}$, $\frac{5}{14}$, $\frac{18}{21}$, and $\frac{6}{28}$. *Ans.* $\frac{4}{14}$, $\frac{5}{14}$, $\frac{12}{14}$, and $\frac{3}{14}$.
9. $\frac{7}{9}$, $\frac{5}{11}$, $\frac{16}{18}$, and $\frac{6}{22}$. *Ans.* $\frac{77}{99}$, $\frac{45}{99}$, $\frac{88}{99}$, and $\frac{27}{99}$.
10. [illegible], and [illegible]. *Ans.* $\frac{39}{143}$, $\frac{55}{143}$, $\frac{65}{143}$, and $\frac{11}{143}$.

11. $\frac{3}{5}$, $\frac{1}{2}$, and $\frac{14}{18}$.
12. $\frac{3}{5}$, $\frac{6}{21}$, and $\frac{14}{21}$.
13. $\frac{1}{2}$, $\frac{7}{3}$, and $\frac{22}{55}$.
14. $\frac{15}{21}$, $\frac{12}{16}$, and $\frac{11}{14}$.
15. $\frac{15}{4}$, $\frac{7}{8}$, and $\frac{8}{18}$.
16. $\frac{3}{11}$, $\frac{2}{14}$, and $\frac{12}{15}$.
17. $\frac{11}{30}$, $\frac{8}{15}$, and $\frac{3}{50}$.
18. $\frac{5}{14}$, $\frac{3}{28}$, and $\frac{13}{42}$.
19. $\frac{3}{10}$, $\frac{19}{30}$, and $\frac{5}{6}$.
20. $\frac{9}{22}$, $\frac{4}{121}$, and $\frac{5}{143}$.
21. $\frac{9}{34}$, $\frac{121}{51}$, and $\frac{5}{68}$.
22. $\frac{17}{16}$, $\frac{11}{128}$, and $\frac{9}{144}$.

7°. *To reduce a complex fraction to a simple one.*

73. Let it be required to reduce $\frac{2\frac{1}{3}}{1\frac{1}{4}}$ to a simple fraction:

OPERATION.

$$\frac{2\frac{1}{3}}{1\frac{1}{4}} = \frac{\frac{7}{3}}{\frac{5}{4}} = \frac{\frac{28}{12}}{\frac{15}{12}} = \frac{28}{15}$$

Explanation.—We first reduce both terms to simple fractions, and then we reduce them to a common denominator; the numerator being 28 times $\frac{1}{12}$ and the denominator 15 times $\frac{1}{12}$, we may strike out the common factor $\frac{1}{12}$, giving $\frac{28}{15}$. In like manner we may treat all similar cases; hence the following

RULE.

Reduce both terms to simple fractions having a common denominator and then suppress the fractional unit.

EXAMPLES.

Reduce the following to simple fractions:

1. $\dfrac{3\frac{1}{2}}{4\frac{5}{8}}$. *Ans.* $\frac{28}{37}$.

2. $\dfrac{\frac{1}{2}\text{ of }\frac{1}{6}}{3\frac{1}{4}}$. *Ans.* $\frac{1}{39}$.

3. $\dfrac{3\frac{1}{2}}{\frac{2}{3}\text{ of }\frac{4}{5}}$. *Ans.* $\frac{105}{16}$.

4. $\dfrac{15}{44\frac{7}{8}}$. *Ans.* $\frac{120}{359}$.

5. $\dfrac{68\frac{3}{4}}{100}$.

6. $\dfrac{42\frac{1}{4}}{13}$.

7. $\dfrac{36\frac{1}{2}}{12\frac{1}{4}}$.

8. $\dfrac{\frac{4}{7}\text{ of }\frac{2}{9}}{63\frac{1}{2}}$.

9. $\dfrac{\frac{2}{3}\text{ of }\frac{1}{2}}{3\frac{1}{4}}$.

10. $\dfrac{22\frac{1}{4}}{44\frac{1}{3}}$.

We have shown in this and the preceding articles that common fractions of all kinds can be reduced to simple fractions. In what follows we shall suppose them to be so reduced.

ADDITION OF FRACTIONS.

Definition.

74. Addition of Fractions is the operation of finding the sum of two or more fractions.

In order that two fractions may be added they must be *similar*, that is, they must have the same integral unit, (Art. **15**).

All abstract fractions are similar.

Operation of Addition.

75. Let it be required to find the sum of $\frac{3}{4}$ and $\frac{5}{6}$.

Explanation.—Having reduced the fractions to the common unit, $\frac{1}{12}$, we see that the unit is taken 9 times in the first and 10 times in the second; hence, the sum of the fractions is equal to the unit $\frac{1}{12}$ taken 9 + 10, or 19 times, that is, to $\frac{19}{12}$, or $1\frac{7}{12}$.

OPERATION.

$$\frac{3}{4}+\frac{5}{6}=\frac{9}{12}+\frac{10}{12}$$
$$=\frac{19}{12}=1\tfrac{7}{12}.$$

In like manner we may treat all similar cases; hence, the following

RULE.

I. Reduce the fractions to simple fractions having a common denominator.

II. Add their numerators for a new numerator, and write the sum over the common denominator.

EXAMPLES.

Add the following groups of fractions:

1. $\frac{7}{15}$, $\frac{10}{21}$, and $\frac{16}{35}$. *Ans.* $\frac{147}{105}=1\frac{2}{5}$.
2. $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{12}$. *Ans.* $\frac{25}{12}=2\frac{1}{12}$.
3. $\frac{3}{5}$, $\frac{2}{7}$, and $\frac{1}{3}$. *Ans.* $\frac{128}{105}=1\frac{23}{105}$.
4. $\frac{2}{5}$, $\frac{3}{8}$, and $\frac{7}{12}$. *Ans.* $1\frac{43}{120}$.
5. $\frac{5}{7}$, $\frac{2}{15}$, and $\frac{9}{35}$. *Ans.* $1\frac{11}{105}$.

6. $\frac{5}{12}$, $\frac{1}{8}$, $\frac{7}{24}$, and $\frac{1}{2}$.
7. $\frac{3}{4}$, $\frac{4}{7}$, and $\frac{7}{9}$.
8. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$.
9. $\frac{3}{6}$, $\frac{15}{7}$, and $\frac{133}{10}$.
10. $\frac{3}{5}$, $\frac{4}{15}$, and $\frac{183}{20}$.
11. $\frac{4}{9}$, $\frac{7}{10}$, $\frac{5}{8}$, and $\frac{17}{18}$.
12. $\frac{6}{7}$, $\frac{5}{14}$, $\frac{11}{21}$, and $\frac{2}{3}$.
13. $\frac{5}{9}$, $\frac{8}{27}$, $\frac{1}{3}$, $\frac{4}{5}$, and $\frac{5}{6}$.
14. $\frac{7}{12}$, $\frac{1}{6}$, $\frac{3}{8}$, $\frac{11}{16}$, and $\frac{1}{2}$.
15. $\frac{4}{7}$, $\frac{9}{14}$, $\frac{4}{5}$, $\frac{2}{21}$, and $\frac{1}{8}$.

When there are mixed numbers, add the sum of the fractional parts to the sum of the whole numbers.

16. $4\frac{1}{2}$, $6\frac{1}{3}$, $2\frac{1}{4}$, and $\frac{5}{6}$. *Ans.* $12 + \frac{23}{12} = 13\frac{11}{12}$.
17. $10\frac{3}{8}$, $7\frac{1}{8}$, $8\frac{2}{9}$, and $16\frac{3}{4}$. *Ans.* $42\frac{17}{36}$.
18. $1\frac{1}{10}$, $6\frac{2}{5}$, $18\frac{19}{20}$, and $2\frac{7}{30}$. *Ans.* $28\frac{41}{60}$.
19. $2\frac{8}{11}$, $6\frac{1}{2}$, and $12\frac{13}{22}$.
20. $67\frac{3}{11}$, $4\frac{9}{33}$, and $600\frac{2}{3}$.
21. $13\frac{1}{7}$, $99\frac{3}{8}$, and $512\frac{5}{14}$.
22. $14\frac{6}{19}$, $3\frac{14}{57}$, and $88\frac{9}{19}$.
23. $900\frac{1}{10}$, $450\frac{37}{100}$, and $6\frac{9}{10}$.
24. $21\frac{1}{4}$, $98\frac{7}{16}$, and $14\frac{11}{24}$.
25. $1\frac{1}{8}$, $4\frac{1}{5}$, and $6\frac{2}{3}$.
26. $4\frac{4}{5}$, $7\frac{5}{12}$, and $8\frac{5}{18}$.
27. $\frac{68}{9}$, $\frac{4}{15}$, and $7\frac{7}{30}$.
28. $5\frac{1}{4}$, $\frac{3}{8}$, and $7\frac{11}{12}$.
29. $4\frac{2}{5}$, $\frac{73}{135}$, and $3\frac{1}{9}$.
30. $2\frac{3}{10}$, $5\frac{11}{15}$, and $6\frac{7}{20}$.
31. $5\frac{5}{11}$, $4\frac{3}{4}$, and $13\frac{1}{8}$.
32. $6\frac{2}{7}$, $11\frac{1}{14}$, and 9.
33. $\frac{11}{12}$, $\frac{5}{14}$, and $2\frac{5}{42}$.
34. $7\frac{1}{4}$, $11\frac{11}{12}$, and $5\frac{4}{9}$.
35. $18\frac{1}{9}$, $2\frac{5}{18}$, and $6\frac{1}{4}$.
36. $\frac{14}{17}$, $\frac{13}{51}$, and $8\frac{1}{51}$.
37. $9\frac{1}{11}$, $\frac{32}{55}$, and $\frac{17}{44}$.
38. $5\frac{4}{7}$, $3\frac{5}{28}$, and $\frac{11}{35}$.
39. $16\frac{1}{5}$, $\frac{21}{25}$, and $\frac{81}{35}$.
40. $8\frac{8}{13}$, $\frac{29}{39}$, and $\frac{52}{65}$.

PRACTICAL PROBLEMS.

1. A farmer has 3 fields; the first contains $31\frac{3}{8}$ *acres*, the second $49\frac{2}{7}$ *acres*, and the third $59\frac{7}{10}$ *acres*; how many acres has he in all? *Ans.* $140\frac{101}{280}$ *acres.*

2. A man earned \$$3\frac{3}{8}$ the first day, \$$4\frac{1}{8}$ the second day, \$$5\frac{5}{16}$ the third day, \$$7\frac{1}{2}$ the fourth day, \$$4\frac{7}{16}$ the fifth day, and \$$3\frac{5}{8}$ the sixth day; how much did he earn in the six days? *Ans.* \$$28\frac{3}{8}$.

3. A. traveled $17\frac{4}{7}$ *miles* the first day, $\frac{3}{4}$ of $17\frac{4}{7}$ *miles* the second day, $22\frac{3}{8}$ *miles* the third day, and $36\frac{1}{4}$ *miles* the fourth day: how far did he travel in the four days?

Ans. $89\frac{3}{8}$ *miles.*

4. B. works $8\frac{3}{5}$ *hours* on Monday, $9\frac{2}{3}$ *hours* on Tuesday, $8\frac{3}{10}$ *hours* on Wednesday, $10\frac{3}{4}$ *hours* on Thursday, $9\frac{4}{5}$ *hours* on Friday, and $10\frac{1}{8}$ *hours* on Saturday: how many hours does he work during the week? *Ans.* $57\frac{29}{120}$ *hours.*

5. A farmer sells 3 tons of hay for \$$47.18\frac{3}{4}$, 3 cows for \$$111.42\frac{1}{4}$, a horse for \$$173.16\frac{1}{6}$, and 100 bushels of oats for \$$62.87\frac{1}{2}$: how much does he receive for the whole? *Ans.* \$$394.64\frac{2}{3}$.

SUBTRACTION OF FRACTIONS.

Definitions.

76. Subtraction of Fractions is the operation of finding the difference between two fractions.

The difference can always be found when the fractions are *similar.*

Operation of Subtraction.

77. Let it be required to find the difference between $\frac{5}{8}$ and $\frac{2}{7}$.

OPERATION.

$$\frac{5}{8} - \frac{2}{7} = \frac{35}{56} - \frac{16}{56} = \frac{19}{56}$$

Explanation.—Having reduced the given fractions to the common unit, $\frac{1}{56}$, we see that the minuend contains this unit 35 times and the subtrahend 16 times; hence, the remainder contains it 35 − 16, or 19 times; the remainder is therefore $\frac{19}{56}$.

Since all similar examples may be treated in the same manner, we have the following

RULE.

I. Reduce the fractions to simple fractions having a common denominator.

II. Subtract the numerator of the subtrahend from that of the minuend for a new numerator, and write the difference over the common denominator.

EXAMPLES.

Find the difference between,

1. $\frac{7}{8}$ and $\frac{1}{2}$. *Ans.* $\frac{3}{8}$.
2. $\frac{7}{12}$ and $\frac{8}{15}$. *Ans.* $\frac{3}{60} = \frac{1}{20}$.
3. $2\frac{3}{5}$ and $1\frac{1}{3}$. *Ans.* $\frac{19}{15} = 1\frac{4}{15}$.
4. $\frac{1}{2}$ of $\frac{4}{5}$ and $\frac{1}{6}$ of $\frac{3}{4}$. *Ans.* $\frac{11}{40}$.

5. $\frac{5}{8}$ and $\frac{1}{4}$.
6. $\frac{4}{9}$ and $\frac{3}{11}$.
7. $\frac{17}{18}$ and $\frac{11}{12}$.
8. $\frac{8}{15}$ and $\frac{9}{20}$.
9. $37\frac{4}{15}$ and $33\frac{5}{24}$.
10. $6\frac{3}{8}$ and $4\frac{1}{2}$.
11. $13\frac{5}{12}$ and $9\frac{4}{12}$.
12. $50\frac{1}{16}$ and $47\frac{1}{24}$.
13. 42 and $30\frac{5}{12}$.
14. $90\frac{8}{11}$ and $25\frac{9}{22}$.
15. $46\frac{5}{8}$ and $15\frac{1}{6}$.
16. $\frac{3}{10}$ and $\frac{2}{9}$ of $\frac{3}{4}$.
17. $98\frac{1}{2}$ and $45\frac{3}{4}$.
18. $150\frac{1}{20}$ and $65\frac{1}{10}$.
19. $\frac{3}{8}$ and $\frac{1}{30}$.
20. $\frac{3}{4}$ and $\frac{1}{10}$ of $2\frac{1}{2}$.
21. $\frac{25}{12}$ and $\frac{13}{24}$.
22. $4\frac{1}{2}$ and $\frac{2}{27}$.
23. $\frac{27}{80}$ and $\frac{5}{24}$.
24. $\frac{5}{14}$ and $\frac{1}{8}$.
25. $8\frac{1}{2}$ and $\frac{4}{9}$.
26. $3\frac{7}{15}$ and $1\frac{2}{3}$.
27. $\frac{7}{25}$ and $\frac{7}{35}$.
28. $\frac{13}{2}$ and $\frac{35}{12}$.
29. 7 and $\frac{2}{11}$.
30. $13\frac{1}{2}$ and $7\frac{11}{12}$.
31. $843\frac{25}{31}$ and $94\frac{3}{4}$.
32. $\frac{1}{2}$ of $\frac{13}{15}$ and $\frac{2}{7}$ of $\frac{9}{11}$.

PRACTICAL PROBLEMS.

1. A. has $\$4\frac{7}{16}$, and B. has $\$3\frac{15}{16}$; how much more has A. than B.? *Ans.* $\$\frac{1}{2}$.

2. A. bought $56\frac{3}{10}$ *pounds* of butter, and sold $\frac{1}{2}$ of it to one person and $13\frac{2}{7}$ *pounds* to another: how much had he left? *Ans.* $14\frac{121}{140}$ *pounds.*

3. A grocer bought 2 *hogsheads* of sugar, each weighing 1,302 *pounds*; he sold $\frac{1}{3}$ of one hogshead and $455\frac{2}{11}$ *pounds* from the other: how many pounds had he remaining? *Ans.* $1{,}714\frac{9}{11}$ *lbs.*

4. A merchant bought 2 pieces of cloth: the first contained $38\frac{3}{8}$ *yards*, and the second $41\frac{4}{7}$ *yards*: he then sold $59\frac{1}{4}$ *yards*; how many yards had he left?

Ans. $20\frac{39}{56}$ *yds.*

5. A man bought a farm of $211\frac{3}{4}$ *acres*, and sold $117\frac{5}{11}$ *acres* of it: how much had he remaining?

Ans. $94\frac{13}{44}$ *acres.*

6. From a cask containing $60\frac{2}{3}$ *gallons* of cider there were drawn off $17\frac{11}{13}$ *gallons*: how much was there left?

Ans. $42\frac{32}{39}$ *gallons.*

7. A sloop has on board $406\frac{3}{5}$ *tons* of coal, of which $311\frac{3}{4}$ *tons* is anthracite, and the remainder cannel: how much cannel does she contain? *Ans.* $94\frac{17}{20}$ *tons.*

8. A merchant had a piece of silk containing $42\frac{1}{4}$ *yards*, from which he sold $17\frac{3}{4}$ *yards*: how much had he remaining? *Ans.* $24\frac{1}{2}$ *yds.*

9. A person having $\$49\frac{3}{4}$, spent $\$4\frac{3}{4}$ in getting to Boston, and $\$5\frac{7}{10}$ in getting to Portland; how much had he left on reaching Portland? *Ans.* $\$39\frac{3}{10}$.

MULTIPLICATION OF FRACTIONS.

Definitions.

78. **Multiplication of Fractions** is the operation of finding the product of two or more fractions.

The operation depends on *principles* 1° *and* 3° of Article **65.**

Operation of Multiplication.

79. Let it be required to multiply $\frac{3}{4}$ by $\frac{5}{7}$.

OPERATION.

$$\frac{3}{4} \times \frac{5}{7} = \frac{3 \times 5}{4 \times 7} = \frac{15}{28}$$

Explanation.—We first multiply $\frac{3}{4}$ by 5, which, according to Principle 1°, gives $\frac{3 \times 5}{4}$; but this result is 7 times the required product, because the multiplier used is 7 times the given multiplier; hence, to find the true product, we must divide it by 7, which, according to Principle 3°, gives $\frac{3 \times 5}{4 \times 7}$, or $\frac{15}{28}$.

In like manner we may treat all similar cases; hence, the following

RULE.

Reduce the factors to simple fractions; then multiply the numerators together for a new numerator, and the denominators for a new denominator.

EXAMPLES.

Perform the following indicated multiplications:

1. $\frac{5}{7} \times \frac{2}{3}$.	*Ans.* $\frac{10}{21}$.
2. $\frac{8}{9} \times \frac{13}{15}$.	*Ans.* $\frac{104}{135}$.
3. $\frac{3}{4} \times \frac{8}{9}$.	*Ans.* $\frac{24}{36} = \frac{2}{3}$.

To multiply a whole number by a simple fraction:

Multiply by the numerator and divide the product by the denominator.

4. $1533 \times \frac{1}{7}$. *Ans.* $\frac{1533}{7} = 219$.

5. $256 \times \frac{3}{11}$. *Ans.* $\frac{256 \times 3}{11} = 69\frac{9}{11}$.

To multiply a whole number by a mixed number:

Multiply first by the fractional part, then by the whole number, and take the sum of the products.

(6.)		(7.)	
928		3)1143	
$6\frac{3}{8}$		$7\frac{1}{3}$	
348	$\frac{3}{8}$ of 928.	381	$\frac{1}{3}$ of 1143.
5568	6 times 928.	8001	7 times 1143.
5916	Product.	8382	Product.

The preceding cases also come under the general rule, the denominator of the whole number being 1.

8. $345 \times 4\frac{7}{9}$. *Ans.* $1648\frac{1}{3}$.

9. $3\frac{7}{9} \times 2\frac{2}{3}$. *Ans.* $\frac{34}{9} \times \frac{8}{3} = \frac{272}{27} = 10\frac{2}{27}$.

The operation of multiplication may often be simplified by indicating all the multiplications and then cancelling the factors common to any numerator and any denominator:

10. $7\frac{1}{2} \times \frac{1}{5}$ of $\frac{4}{9}$.

$$\textit{Ans. } \frac{15}{2} \times \frac{1}{5} \times \frac{4}{9} = \frac{\cancel{3} \times \cancel{5}}{\cancel{2}} \times \frac{1}{\cancel{5}} \times \frac{\cancel{2} \times 2}{\cancel{3} \times 3} = \frac{2}{3}.$$

11. $\frac{1}{3}$ of $\frac{2}{3} \times \frac{43}{8}$ of 3. *Ans.* $5\frac{1}{3}$.

12. [illegible] × $\frac{8}{9}$.

13. [illegible] × [illegible].

14. [illegible] × [illegible].

15. $\frac{8}{9} \times \frac{21}{160}$.

16. $\frac{80}{9} \times \frac{4}{7}$.

17. $\frac{7}{8} \times 12\frac{2}{3}$.

18. $7\frac{1}{2} \times 8\frac{3}{4}$.

19. $7\frac{1}{3} \times 61\frac{3}{5}$.

20. $\frac{2}{3}$ of $7 \times \frac{3}{5}$.

21. $15\frac{5}{7} \times \frac{21}{5}$.

22. $6\frac{2}{3} \times \frac{169}{16}$.

23. $\frac{6}{11}$ of $\frac{2}{3} \times \frac{19}{24}$.

24. $\frac{3}{4}$ of $7\frac{1}{5} \times \frac{1}{4}$ of 90.

25. $\dfrac{7}{10} \times \dfrac{8\frac{1}{2}}{14}$.

The rule may be extended to the multiplication of any number of factors:

26. $\frac{2}{3} \times \frac{3}{5} \times \frac{7}{11}$. *Ans.* $\dfrac{2 \times \cancel{3} \times 7}{\cancel{3} \times 5 \times 11} = \dfrac{14}{55}$.

27. $3\frac{1}{2} \times \frac{7}{8} \times 11\frac{1}{2}$. *Ans.* $\frac{1127}{32} = 35\frac{7}{32}$.

28. $\frac{7}{18} \times \frac{3}{14} \times \frac{36}{5}$.

29. $3\frac{1}{8} \times \frac{20}{187} \times \frac{34}{5}$.

30. $3\frac{1}{3} \times 7\frac{1}{2} \times \frac{27}{32}$.

31. $\frac{61}{17} \times 2\frac{1}{9} \times 1\frac{15}{19}$.

32. $5\frac{1}{2} \times \frac{105}{308} \times 3\frac{1}{5}$.

33. $2\frac{2}{9} \times 5\frac{2}{5} \times \frac{122}{355}$.

34. $114\frac{113}{216} \times 81\frac{1}{2}$.

35. $\frac{3}{4}$ of $\frac{5}{11} \times \frac{17}{29}$.

36. $2\frac{1}{8} \times \frac{50}{51} \times \frac{3}{5}$.

37. $\frac{7}{15}$ of $3\frac{1}{3} \times \frac{15}{17}$.

38. $\frac{11}{15} \times \frac{7}{9} \times 3\frac{1}{5}$.

39. $217\frac{1}{7} \times 112\frac{3}{5}$.

40. $\frac{16}{21} \times \frac{7}{8} \times \frac{3}{17}$.

41. $8\frac{1}{4} \times 8\frac{1}{4} \times 8\frac{1}{4}$.

PRACTICAL PROBLEMS.

1. If a man earns $\$33\frac{1}{3}$ per week, how much will he earn in a year of 52 weeks? *Ans.* $\$1,733\frac{1}{3}$.

2. A farmer bought 43 acres of land at $\$104\frac{3}{4}$ per acre, 16 cows at $\$28\frac{5}{8}$ each, and 2 plows at $\$11\frac{6}{13}$ each; what did they all cost him? *Ans.* $\$4,985\frac{9}{52}$.

3. A. bought $319\frac{2}{7}$ acres of land at \$200 per acre; he then sold $250\frac{4}{9}$ acres at \$250 per acre, and the remainder at $\$266\frac{2}{3}$ per acre; how much did he gain?

Ans. $\$17,111\frac{121}{189}$.

4. What must be paid for 600 barrels of flour at \$5.37½ per barrel? *Ans.* \$3,225.

5. A grocer bought 100 barrels of flour at \$6⅞ per barrel: he sold 49 barrels at \$7½ per barrel and the rest at \$7⅛ per barrel; how much did he gain?

Ans. 43⅜.

6. What is the cost of 33⅓ lbs. of tea at 93¾ cents a pound? *Ans.* \$31.25.

DIVISION OF FRACTIONS.

Definition.

80. **Division of Fractions** is the operation of finding the quotient of one fraction by another.

The operation depends on *principles* 1° *and* 3° of Article **65.**

Operation of Division.

81. Let it be required to find the quotient of $\frac{3}{4}$ by $\frac{5}{7}$:

OPERATION.

$$\frac{3}{4}\div\frac{5}{7}=\frac{3\times7}{4\times5}=\frac{21}{20}$$

Explanation.—In the first place we divide $\frac{3}{4}$ by 5, which, according to principle 3°, gives $\frac{3}{4\times5}$; this result is only one-seventh of the required quotient, because the divisor used is 7 times the given divisor; hence, to obtain the true quotient we must multiply it by 7; performing the multiplication, in accordance with principle 1°, we have for the required quotient $\frac{3\times7}{4\times5}$, or $\frac{21}{20}$. Here we have inverted the divisor, that is, we have made its terms change places and then we have proceeded as in multiplication.

In like manner we may treat all similar cases; hence the following

RULE.

Reduce both dividend and divisor to simple fractions; then invert the divisor and proceed as in multiplication.

EXAMPLES.

Perform the following indicated divisions:

1. $\frac{2}{11}$ by $\frac{3}{5}$. *Ans.* $\frac{2 \times 5}{11 \times 3} = \frac{10}{33}$.

2. $4\frac{1}{3}$ by $2\frac{3}{4}$. *Ans.* $\frac{13 \times 4}{3 \times 11} = \frac{52}{33} = 1\frac{19}{33}$.

3. $\frac{1}{2}$ of $\frac{3}{5}$ by $\frac{7}{11}$. *Ans.* $\frac{3 \times 11}{10 \times 7} = \frac{33}{70}$.

4. $\frac{5}{8} \div \frac{1}{4}$.

5. $\frac{51}{64} \div \frac{17}{16}$.

6. $\frac{8}{9} \div \frac{13}{18}$.

7. $\frac{16}{45} \div \frac{14}{15}$.

8. $\frac{13}{20} \div \frac{3}{5}$.

9. $\frac{13}{24} \div \frac{7}{12}$.

10. $\frac{45}{32} \div \frac{15}{64}$.

11. $\frac{55}{51} \div \frac{3}{4}$.

12. $\frac{401}{500} \div \frac{401}{7}$.

13. $\frac{14}{15} \div \frac{7}{1}$.

14. $\frac{28}{37} \div \frac{4}{1}$.

15. $\frac{44}{1} \div \frac{11}{12}$.

16. $3\frac{3}{4} \div 5$.

17. $54 \div 3\frac{3}{8}$.

18. $611\frac{7}{11} \div 20\frac{7}{11}$.

19. $100\frac{27}{30} \div 66\frac{2}{3}$.

20. $\frac{16}{17} \div 17\frac{1}{17}$.

21. $16\frac{1}{2} \div 8\frac{1}{4}$.

22. $1\frac{1}{2} \div 3\frac{7}{8}$.

23. $2\frac{1}{2} \div 1\frac{2}{3}$.

24. $8\frac{1}{3} \div 12\frac{2}{9}$.

25. $4\frac{1}{2} \div \frac{1}{10}$.

26. $2\frac{5}{8} \div 7$.

27. $9 \div 6\frac{3}{5}$.

28. $\frac{1}{3}$ of $\frac{5}{7} \div \frac{3}{8}$ of $\frac{1}{5}$.

29. $2\frac{3}{7} \div 8\frac{8}{9}$.

30. $(8\frac{1}{4} + 3\frac{1}{2}) \div 7\frac{3}{5}$.

31. $3\frac{3}{4} \div (1\frac{1}{3} + 2\frac{1}{5})$.

32. $4\frac{1}{5} \div \frac{5}{17}$.

33. $\frac{32}{35} \div 2\frac{1}{2}$.

PRACTICAL PROBLEMS.

1. What number taken from $14\frac{3}{8}$ will leave $1\frac{3}{4}$?

Ans. $12\frac{5}{8}$.

2. What number multiplied by $1\frac{3}{8}$ will give $14\frac{3}{4}$?

Ans. $10\frac{8}{11}$.

3. The difference of two numbers is $15\frac{4}{35}$ and the greater number is $20\frac{11}{15}$; what is the less number?

Ans. $5\frac{13}{21}$.

4. A man inherits $\frac{3}{8}$ of an estate and gives his son $\frac{1}{2}$ of his share; what part of the estate does the son receive? *Ans.* $\frac{3}{16}$.

5. If $37\frac{1}{2}$ ounces of silver cost $\$31\frac{1}{4}$, what does 1 ounce cost, and what do 150 ounces cost?

Ans. $\$\frac{5}{6}$; and $125.

6. If $3\frac{3}{4}$ bushels of wheat cost $\$2\frac{3}{8}$, what does 1 bushel cost? *Ans.* $\$\frac{19}{30}$.

7. If a man travels $401\frac{1}{2}$ miles in $13\frac{1}{4}$ days, how many miles does he travel per day? *Ans.* $30\frac{16}{53}$.

8. A. divides $\$3,000\frac{3}{7}$ into 7 equal shares and gives $4\frac{1}{2}$ of these shares to a benevolent society; how much does he give to it? *Ans.* $\$1,928\frac{83}{98}$.

9. A. can build a wall in 10 days, B. can do it in 12 days, and C. can do it in 15 days; what part of the wall can they all build in 1 day?

Ans. $\frac{1}{10} + \frac{1}{12} + \frac{1}{15} = \frac{1}{4}$.

10. How long will it take them all to build the wall?

Ans. 4 *days.*

11. If 13 yards of muslin cost $\$\frac{2}{25}$, what will 1 yard cost? *Ans.* $\$\frac{2}{325}$.

12. If $\frac{7}{8}$ of a ton of coal costs \$13, what will 1 ton cost? *Ans.* \$$14\frac{6}{7}$.

13. If coffee costs $13\frac{3}{4}$ cents a pound, how much can be bought for \$10. *Ans.* $72\frac{8}{11}$ *lbs.*

14. How many pounds of coffee can be bought for \$$78\frac{5}{7}$ at \$$\frac{3}{16}$ per pound? *Ans.* $419\frac{17}{21}$ *lbs.*

15. A. can do a piece of work in 3 days and B. can do it in 2 days; how long will it take them both to do it? *Ans.* $1\frac{1}{5}$ *days.*

16. If $3\frac{3}{4}$ bushels of oats cost \$$2\frac{3}{8}$, what will 2 bushels cost? *Ans.* \$$1\frac{4}{15}$.

Simplifications.

82. The application of the rules for multiplication and division of fractions leads to certain simplifications in the operations of multiplication and division of whole numbers, of which the following are some of the most important.

1°. *To multiply any number by* 25:

This is equivalent to multiplying by the fraction $\frac{100}{4}$; hence, the following

RULE.

Annex two ciphers and divide by 4.

EXAMPLES.

1. Multiply 394 by 25. *Ans.* $\frac{39400}{4} = 9,850$.
2. Multiply 3,724 by 25. *Ans.* 93,100.
3. Multiply 8,123 by 25. *Ans.* 203,075.

2°. *To multiply any number by* $12\frac{1}{2}$:

This is equivalent to multiplying by $\frac{100}{8}$; hence, the following

RULE.

Annex two ciphers and divide by 8.

EXAMPLES.

4. Multiply 81 by $12\frac{1}{2}$. *Ans.* $\frac{8100}{8} = 1012\frac{1}{2}$.
5. Multiply 914 by $12\frac{1}{2}$. *Ans.* 11,425.
6. Multiply 4,834 by $12\frac{1}{2}$. *Ans.* 60,425.

In like manner to multiply by 125 we annex three ciphers and divide by 8.

7. Multiply 81 by 125. *Ans.* $\frac{81000}{8} = 10{,}125$.
8. Multiply 73 by 125. *Ans.* 9,125.

3°. *To multiply any number by* $33\frac{1}{3}$:

This is equivalent to multiplying by $\frac{100}{3}$; hence, the following

RULE.

Annex two ciphers and divide by 3.

EXAMPLES.

9. Multiply 375 by $33\frac{1}{3}$. *Ans.* $\frac{37500}{3} = 12{,}500$.
10. Multiply 28,452 by $33\frac{1}{3}$. *Ans.* 948,400.

To multiply by $333\frac{1}{3}$ we annex three ciphers and divide by 3.

11. Multiply 315 by $333\frac{1}{3}$. *Ans.* $\frac{315000}{3} = 105{,}000$.
12. Multiply 45 by $333\frac{1}{3}$. *Ans.* 15,000.

By reversing the preceding rules, we have the following:

4°. *To divide a number by 25:*

RULE.

Multiply by 4 and divide by 100.

EXAMPLES.

13. Divide 9,850 by 25. *Ans.* $\frac{9850 \times 4}{100} = 394.$

14. Divide 93,100 by 25. *Ans.* 3,724.

15. Divide 87,525 by 25. *Ans.* 3,501.

16. Divide 46,350 by 25. *Ans.* 1,854.

17. Divide 174,025 by 25. *Ans.* 6,961.

5°. *To divide any number by* $12\frac{1}{2}$:

RULE.

Multiply by 8 and divide by 100.

EXAMPLES.

18. Divide $1,012\frac{1}{2}$ by $12\frac{1}{2}$. *Ans.* $\frac{1012\frac{1}{2} \times 8}{100} = 81.$

19. Divide 11,425 by $12\frac{1}{2}$. *Ans.* 914.

20. Divide 9,125 by $12\frac{1}{2}$. *Ans.* 730.

6°. *To divide any number by* $33\frac{1}{3}$:

RULE.

Multiply by 3 and divide by 100.

EXAMPLES.

21. Divide 12,500 by 33⅓. *Ans.* $\frac{12500 \times 3}{100} = 375.$

22. Divide 29,100 by 33⅓. *Ans.* 873.

23. Divide 8,700 by 33⅓. *Ans.* 261.

Corresponding methods may be employed in many similar cases.

EXAMINATION QUESTIONS.

(60.) What is a fractional unit? What is a half, a third, a fourth, &c.? What is the reciprocal of a number? **(61.)** What is a fraction? How do you write a common fraction? What is the denominator? the numerator? What are terms? **(63.)** In how many ways may we regard a fraction? Illustrate. **(64.)** What is a proper fraction? Illustrate. An improper fraction? Illustrate. A mixed number? Illustrate. A simple fraction? Illustrate. A complex fraction? Illustrate. A compound fraction? Illustrate. **(65.)** State the fundamental principles of fractions. **(66.)** What is reduction? **(67.)** Give the rule for reducing a whole number to a fraction with a given unit. **(68.)** Give the rule for reducing a mixed number to a fraction. **(69.)** Give the rule for reducing an improper fraction to a mixed number. **(70.)** Give the rule for reducing a fraction to its lowest terms. **(71.)** Give the rule for reducing a compound fraction to a simple one. **(72.)** Give the rule for reducing fractions to a common denominator. **(73.)** Give the rule for reducing a complex fraction to a simple one. **(74.)** What is addition of fractions? **(75.)** Give the rule for addition of fractions. **(76.)** What is subtraction of fractions? **(77.)** Give the rule for subtraction of fractions. **(78.)** What is multiplication of fractions? **(79.)** Give the rule for the multiplication of fractions. **(80.)** What is division of fractions? **(81.)** Give the rule for division of fractions. **(82.)** Give the simplified rule for multiplying by 25; by 12½; by 125; by 33⅓. Give the simplified rulo for dividing by 25; by 12½; by 33⅓.

II. DECIMAL FRACTIONS.

Definitions.

83. A *power* is a product obtained by taking a number two or more times as a factor. The number taken is then said to be its own *first* power. Thus,

10 is called the *first* power of 10;

10 × 10, or 100, is the *second* power of 10;

10 × 10 × 10, or 1000, is the *third* power of 10;

and so on. The number of ciphers in any power of 10 is indicated by the name of the power. Thus, the *seventh* power of 10 contains 7 ciphers, that is, it is 10,000,000.

Decimals.

84. A **Decimal Fraction** is one whose denominator is some power of 10, as $\frac{3}{10}$, $\frac{4}{100}$, $\frac{15}{1000}$, etc.

The denominators may be *written*, as shown above, or they may be *indicated* by a point, (.). In the former case they do not differ from common fractions; in the latter case they are called *decimals*, and the point is called the *decimal point*.

Notation of Decimals.

85. A *decimal* fraction may be expressed by writing its numerator and then placing a decimal point so that the number of figures following it shall be equal to the number of ciphers in the denominator. Thus, $\frac{3}{10} = .3$, $\frac{14}{100} = .14$, $\frac{125}{1000} = .125$.

If the number of ciphers in the denominator is greater than the number of figures in the numerator, the requisite number of ciphers must be *prefixed to*, that is, written before, the numerator. Thus, $\frac{3}{100} = .03$, $\frac{2}{1000} = .002$.

If the number of ciphers in the denominator is less than the number of figures in the numerator, the result will consist of an integral part and a decimal. Thus, $\frac{118}{10} = 11.8$, $\frac{1234}{100} = 12.34$, $\frac{1367}{1000} = 1.367$. Such expressions are called *mixed* decimals.

Decimal places are counted from the decimal point toward the right. The first figure after the decimal point stands in the *first* place of decimals, the next figure stands in the *second* place, the next in the *third* place, and so on.

EXAMPLES.

Express the following fractions as decimals:

1. $\frac{82}{100}$. *Ans.* .82.
2. $\frac{314}{1000}$. *Ans.* .314.
3. $\frac{51}{1000}$. *Ans.* .051.
4. $\frac{3}{10000}$. *Ans.* .0003.
5. $\frac{674}{100}$. *Ans.* 6.74.
6. $\frac{372}{10}$. *Ans.* 37.2.
7. $\frac{114}{10000}$. *Ans.* .0114.
8. $\frac{18}{100000}$. *Ans.* .00018.

Numeration of Decimals.

86. Decimals may be read *as common fractions*, or they may be read *by periods*, as explained in Articles **11** and **12**.

1°. *As Common Fractions.*—In this case we disregard the decimal point, and read the given figures for a numerator; we then supply the denominator, remembering that it is equal to 1 followed by as many ciphers as there are figures in the given decimal. Thus, the decimal .014 is equivalent to $\frac{14}{1000}$, and may be read fourteen *one-thousandths;* the mixed decimal 7.04 may be read seven *units* and four *one-hundredths,* or seven hundred and four *one-hundredths.*

EXAMPLES.

Read the following decimals:

1. .0345.
Ans. Three hundred and forty-five *ten-thousandths.*

2. .00214.
Ans. Two hundred and fourteen *hundred-thousandths.*

3. .000052. *Ans.* Fifty-two *millionths.*

In mixed decimals we read the entire part, adding the word *units,* and then we read the decimal part as just explained.

4. 18.032.
Ans. Eighteen *units,* and thirty-two *thousandths.*

5. 125.000123.
Ans. One hundred and twenty-five *units,* and one hundred and twenty-three *millionths.*

2°. *By Periods.*—In this case, commencing at the decimal point, we separate the decimal into periods of three figures each, annexing ciphers, if necessary, to complete the last period. These periods are named as shown below, and are read as explained in Article **12**.

Extension of the Numeration Table.

Millions,				Thousands,				Units.				Thousandths,				Millionths,				Billionths.		
hundreds of	tens of	units of		hundreds of	tens of	units of		hundreds of	tens of	units of		hundreds of	tens of	units of		hundreds of	tens of	units of		hundreds of	tens of	units of
1	8	7	,	3	1	9	,	3	2	8	.	3	8	2	,	5	6	7	,	8	0	0

The number written above is read, 187 *millions*, 319 *thousands*, 328 *units*, and 382 *thousandths*, 567 *millionths*, 800 *billionths*. The periods that follow *billionths* are *trillionths*, *quadrillionths*, *quintillionths*, etc.

EXAMPLES.

Read the following pure and mixed decimals:

1. .035,679,500.

Ans. 35 *thousandths*, 679 *millionths*, and 500 *billionths*.

2. .000,067,35. *Ans.* 67 *millionths*, and 350 *billionths*.

3. 87.007,2.

Ans. 87 *units*, 7 *thousandths*, and 200 *millionths*.

4. 3.141,592,653,589,793.

Ans. 3 *units*, 141 *thousandths*, 592 *millionths*, 653 *billionths*, 589 *trillionths*, and 793 *quadrillionths*.

It will be seen from what precedes that decimals may be written and read in the same manner as whole numbers; this should be the case, since they are formed in accordance with the same general laws.

Fundamental Principles.

87. Removing the decimal point one place to the right does not change the value of the numerator, but it divides the denominator by 10; this, according to Principle 4°, Article **65**, is equivalent to multiplying the decimal by 10; hence, the following principle:

1°. *Moving the decimal point one place to the right is equivalent to multiplying the decimal by 10.*

In like manner we have the following principle:

2°. *Moving the decimal point one place to the left is equivalent to dividing the decimal by 10.*

Annexing a cipher to a decimal multiplies both numerator and denominator by 10; but this does not affect the value of the fraction, (Art. **65**, Principle 5°); hence, the following principle:

3°. *Annexing a cipher to a decimal does not change its value.*

In like manner we may deduce the following principle:

4°. *Striking out a terminal cipher does not change the value of a decimal.*

Changes of Form.

88. Decimals may be reduced to common fractions and common fractions to decimals. These reductions depend on principles already explained.

1°. *To reduce a decimal to a common fraction.*

89. In accordance with the explanation given in Article **86**, we have the following

RULE.

Write the decimal, omitting the decimal point and prefixed ciphers, for a numerator; and for a denominator write 1 followed by as many ciphers as there are figures in the given decimal.

EXAMPLES.

Reduce the following decimals to common fractions:

1. .125.	*Ans.* $\frac{125}{1000}=\frac{1}{8}$.
2. .0365.	*Ans.* $\frac{365}{10000}=\frac{73}{2000}$.
3. .00024.	*Ans.* $\frac{24}{100000}=\frac{3}{12500}$.

In reducing mixed decimals to the required form the entire part remains unchanged.

4. 32.45.	*Ans.* $32\frac{45}{100}=32\frac{9}{20}$.
5. 3.0016.	*Ans.* $3\frac{16}{10000}=3\frac{1}{625}$.
6. 2.014.	*Ans.* $2\frac{7}{500}$.
7. 2.22.	*Ans.* $2\frac{11}{50}$.
8. 1.103.	*Ans.* $1\frac{103}{1000}$.

2°. *To reduce a common fraction to a decimal.*

90. Let it be required to reduce $\frac{5}{8}$ to a decimal.

OPERATION.

$$\begin{array}{r} 8)5000 \\ \hline .625 \end{array}$$

Explanation.—The value of $\frac{5}{8}$ is equal to $5 \div 8$, (Art. **62**); to find this quotient we annex three ciphers to 5, which is equivalent to multiplying it by 1,000, and then perform the division; but this result is 1,000 times the true value of the fraction; we therefore divide it by 1,000, which is done by pointing off three decimal figures.

In like manner we may treat all similar cases; hence, the following

RULE.

Annex ciphers to the numerator and divide the result by the denominator; then point off from the right of the quotient a number of decimal figures equal to the number of ciphers annexed.

EXAMPLES

Reduce the following fractions to decimals:

1. $\frac{37}{125}$. *Ans.* .296.
2. $\frac{56}{25}$. *Ans.* 2.24.
3. $\frac{23}{32}$. *Ans.* .71875.

If the number of figures in the quotient is less than the number of ciphers annexed, prefix the requisite number of ciphers.

4. $\frac{4}{125}$. *Ans.* .032.
5. $\frac{3}{128}$. *Ans.* .0234375.

The given fraction should be reduced to its lowest terms before the rule is applied. Then, if the denominator contains any prime factor except 2 or 5 the division will not terminate, and the decimal obtained will be an *approximate* value of the given fraction.

6. $\frac{4}{21}$. *Ans.* .1904.

In the preceding example $\frac{4}{21}$ is less than $\frac{1905}{10000}$ and greater than $\frac{1904}{10000}$; hence, it differs from either by less than they differ from each other, that is, by less than $\frac{1}{10000}$.

In like manner it may be shown that the approximate value obtained by stopping at any decimal figure is in error by less than the corresponding fractional unit.

In the preceding example the decimal figure following 4 is 7; hence, the true value of the fraction is nearer to .1905 than it is to .1904, that is, .1905 differs from the true value by less than $\frac{1}{2}$ of $\frac{1}{10000}$, or by less than $\frac{1}{20000}$.

If we stop at any decimal figure and increase it by 1 when the figure next following is equal to or greater than 5, the error will always be less than $\frac{1}{2}$ the corresponding fractional unit.

Find the values of the following fractions to within less than $\frac{1}{20000}$:

7. $\frac{29}{39}$. *Ans.* .7436.
8. $\frac{3}{4}$ of $\frac{1}{7}$. *Ans.* .1071.
9. $\dfrac{35}{6.41}$. *Ans.* 5.4602.
10. $\frac{19}{37}$. *Ans.* 0.5135.
11. $\frac{15}{21}$. *Ans.* 0.7143.
12. $\frac{4}{21}$. *Ans.* 0.1905.

13. $3\frac{14}{17}$.
14. $4\frac{10}{11}$.
15. $\frac{1}{2}$ of $3\frac{1}{4}$.
16. $\frac{3}{11}$ of $7\frac{1}{5}$.
17. $\frac{2}{3}$ of $\frac{5}{9}$ of $4\frac{1}{5}$.
18. $21\frac{21}{32}$.
19. $1\frac{22}{225}$.
20. $14\frac{14}{15}$.
21. $13\frac{11}{117}$.
22. $4\frac{418}{535}$.

ADDITION OF DECIMALS.

Definition.

91. Addition of Decimals is the operation of finding the sum of two or more decimals.

The operation depends on the same principles as the addition of simple numbers, (Art. **19**).

Operation of Addition.

92. Let it be required to find the sum of 4.035, 76.19, 114.0305:

Explanation.—The decimals are written so that units of the same order stand in the same column; beginning at the right, each column is added separately, *setting down* and *carrying* as in the addition of simple numbers. Hence, the following

OPERATION.

	4.035
	76.19
	114.0305
Sum.	194.2555

RULE.

Write the decimals so that units of the same order shall stand in the same column, and add as in simple numbers.

EXAMPLES.

(1.)	(2.)	(3.)	(4.)
3.057	5.6000	5.43	0.105
14.086	17.0032	12.998	0.0012
209.3154	35.9070	317.0971	0.25
226.4584	58.5102	335.5251	0.3562

Find the sums of the following groups of decimals:

5. .632, .718, 3.202, and 111.1. *Ans.* 115.652.
6. .0049, 47.0426, 37.041, and 360.0039. *Ans.* 444.0924.
7. 3.25, 42,348, 748.4, and 29.32. *Ans.* 823.318.
8. 672.5, 4.923, 80, and .0764.
9. 72.5, 140, 340.03, 21.5715, and 4.0008.
10. 2.8146, .0938, 8.875, 231.2788, and 4.0087.
11. 12.3004, 6732.56, 4570.0004, and 1.32.
12. 54.3, 7.29, 180.0046, 187, and 3.024.
13. 57.038. 95.00487, 53.4690, and 107.00003.
14. 27.5037, 342.042, and 86.9735.
15. 62.7, 2.03, 4.009, 78.15, and 114.
16. 4.005, .0032, .4, and 3.275.
17. .0009, 3.0021, .128. 8.0469, and 59.
18. 3.0102, 11.5008, 73.07, 2.92, and 9.5.
19. 2.005, 110.301, .069, 7.375. and 2.25.
20. 17.215, 3.0567, 2.072, 4.009, and 54.75.
21. 231.8, 45.001, 2.7169. 11.007, and 6.055.
22. 29.157, 8.0016, 77.29, 32.004, and 8.845.
23. 3.29, 15.671, .0053, and 22.67.
24. 14.2351, 651.012, 2.219, 3.157, and 13.614.
25. 213.7, 2.913, 14.769, and .0078.
26. 15.753, 2.069, 17.6143, and 3.2107.

SUBTRACTION OF DECIMALS.

Definitions.

93. **Subtraction of Decimals** is the operation of finding the difference between two decimals.

The operation depends on the same principles as the subtraction of simple numbers, (Art. **25**).

Operation of Subtraction.

94. Let it be required to subtract 4.079 from 11.362.

Explanation.—The subtrahend is written under the minuend, so that units of the same order shall stand in the same column; the operation is then performed as in the subtraction of simple numbers. Hence, the following

OPERATION.

Minuend,	11.362
Subtrahend,	4.079
Remainder,	7.283

RULE.

Write the subtrahend under the minuend, so that units of the same order shall stand in the same column; then subtract as in simple numbers.

EXAMPLES.

	(1.)	(2.)	(3.)
Minuend,	5.316	17.0091	1075.0567
Subtrahend,	2.013	11.9902	287.9374
Remainder,	3.303	5.0189	787.1193

If the subtrahend contains more decimal figures than the minuend, annex the requisite number of ciphers to the minuend, or conceive them to be annexed, (Principle 3°, Art. **87**).

(4.)	(5.)	(6.)	(7.)
13.700	13.7	884.1300	884.13
8.299	8.299	33.7865	33.7865
5.401	5.401	850.3435	850.3435

8. From 298.789 subtract 196.493. *Ans.* 102.296.

9. From 2684.11 subtract 199.8637. *Ans.* 2484.2463.

10. From 127.334 subtract 55.827. *Ans.* 71.507.

11. From 94.8607 subtract 27.861. *Ans.* 66.9997.

12. From 986.444 subtract 98.6438. *Ans.* 887.8002.

13. From 17.02 take 7.25. *Ans.* 9.77.

14. From 2.867 take .9965. *Ans.* 1.8705.

15. From 661 subtract 95.072. *Ans.* 565.928.

16. From 25000 subtract 1.077. *Ans.* 24998.923.

17. From 320000.0045 subtract 79807.0006.
Ans. 240193.0039.

18. From 1 take .001.

19. From 10 subtract .0003.

20. What is the difference between 6 and .0006?

21. From 14.003 subtract 9.875.

22. From 13.4072 subtract 9.1875.

23. From 18.65 take 12.0734.

24. From 17.314 *yds.* subtract 12.9921 *yds.*

25. From $13.3125 subtract $8.4139.

26. From 21.007 *lbs.* subtract 9.334 *lbs.*

27. From 184.073 *ft.* subtract 110.823 *ft.*

28. From 333.756 *yds.* subtract 111.995 *yds.*

29. From 87.007 *lbs.* subtract 10.895 *lbs.*

30. From $34.883 subtract $9.43.

MULTIPLICATION OF DECIMALS.

Definition.

95. **Multiplication of Decimals** is the operation of finding the product of two decimals.

The operation depends on the same principles as the multiplication of simple numbers, (Art. **31**), and also on the rule for the multiplication of common fractions, (Art. **79**).

Operation of Multiplication.

96. Let it be required to find the product of 7.8 and .82.

OPERATION.

$$7.8 \times .82 = \frac{78}{10} \times \frac{82}{100}$$

$$= \frac{6396}{1000} = 6.396.$$

Explanation. — Here the decimals are first reduced to equivalent common fractions and multiplied together by the rule of Article **79**; the resulting fraction is then reduced to the decimal form; in doing this, we have actually multiplied the given decimals together, without reference to their decimal points, and in the result pointed off as many decimal figures as there are in both factors.

Since all similar cases may be treated in the same manner, we have the following

RULE.

Multiply as in simple numbers, and point off, from the right of the product, as many decimal figures as there are in both factors.

EXAMPLES.

Perform the following indicated multiplications:

1. 3.05 × 4.102. *Ans.* 12.5111.
2. 38.4 × 16.7. *Ans.* 641.28.
3. 14.25 × .375. *Ans.* 5.34375.
4. 1500 × .00014. *Ans.* .21.

If the number of places in the product is less than the number of places in both factors, prefix as many ciphers as may be necessary.

5. .003 × .042. *Ans.* 0.000126.

6. 1.009 × .0012. *Ans.* 0.0012108.
7. 146.05 × 128.6. *Ans.* 18782.03.
8. .565 × .16. *Ans.* 0.0904.
9. .0463 × .0081. *Ans.* 0.00037503.

Find the following products to within $\frac{1}{20000}$:

10. 701.005 × 60.06.
11. 456.05 × 3.825.
12. 308.25 × .0775.
13. 27.032 × 14.3.
14. 380.06 × 22.
15. 24.07 × .125.
16. .75 × .33.
17. 456.87 × .066.
18. 798.007 × .08.
19. .034 × .08.
20. 7.45 × 2.7504.
21. 42.2 × 2.004.
22. 79.004 × .00473.
23. 412.5384 × 1.00003.
24. 40.86 × .00293.
25. 0.0756 × 6.75.
26. .2897 × 3020.
27. 37.55 × 45.64.

To multiply by 10, 100, 1,000, etc., move the decimal point as many places to the right as there are ciphers in the multiplier, annexing ciphers to the multiplicand if necessary.

28. 375.456 × 10. *Ans.* 3754.56.
29. 34.186 × 100. *Ans.* 3418.6.
30. .0075 × 1,000. *Ans.* 7.5.
31. 3.05 × 10,000. *Ans.* 30,500.
32. 7.0356 × 100. *Ans.* 703.56.

Find the following continued products to within $\frac{1}{20000}$:

33. 3.005 × 21.82 × 14.71.
34. 8.013 × 11.7 × 0.774.
35. 12.12 × 300.7 × 8.004.
36. 0.713 × 2.346 × 2.005.

37. $37.416 \times 20.23 \times 67.45 \times 2.2$.

38. $2.005 \times 6.143 \times 33.333$.

39. $4.4444 \times 5.5555 \times 6.23 \times .5$.

40. $3.3333 \times 4.2222 \times 8.07 \times 4.3$.

In the preceding examples we suppose the multiplications to be performed in the order given, and that each successive product is taken to within less than $\frac{1}{20000}$, as explained in Article **90**.

DIVISION OF DECIMALS.

Definition.

97. **Division of Decimals** is the operation of finding the quotient of one decimal by another.

The operation depends on the same principles as the division of simple numbers, (Art. **38**), and also on the rule for the division of common fractions, (Art. **81**).

Operation of Division.

98. Let it be required to divide 7.8 by .125.

OPERATION.

$$\frac{7.8}{.125} = \frac{78}{10} \div \frac{125}{1000}$$

$$= \frac{78}{10} \times \frac{1000}{125} = \frac{78000}{125} \div 10$$

$$= 624 \div 10 = 62.4$$

Explanation.—Here we have reduced the given decimals to common fractions and divided by the rule of Art. 81; we have then reduced the result to the decimal form; in doing this, we have actually annexed three decimal ciphers to the dividend, (Art. 87), and divided the result by the divisor, without reference to the decimal point; then from the quotient we have pointed off as many decimal figures as the number in the reduced dividend exceeds that in the divisor.

All similar cases may be treated in the same manner; hence, the following

RULE.

Annex as many decimal ciphers to the dividend as necessary; then divide as in simple numbers and point off from the right of the quotient as many decimal figures as the number of such figures in the reduced dividend exceeds that in the divisor.

EXAMPLES.

Perform the following indicated divisions:

1. 9.36 ÷ 3.9. *Ans.* 2.4.
2. 40.05 ÷ 4.5. *Ans.* 8.9.

The reduced dividend *must* contain as many decimal places as the divisor, but it *may* contain more.

3. .53 ÷ .0053. *Ans.* 100.
4. .0141 ÷ .00047. *Ans.* 30.

If the number of decimal figures in the quotient is less than is required by the rule, a sufficient number of ciphers must be prefixed.

5. 2.3 ÷ 1437.5. *Ans.* .0016.
6. .00125 ÷ .5. *Ans.* .0025.
7. 34.75 ÷ 25. *Ans.* 1.39.
8. 46.103 ÷ 2.14. *Ans.* 21.5434.
9. 346.7007 ÷ .909.
10. 3414.52 ÷ 30.25.
11. 16.025 ÷ .045.
12. .9375 ÷ .075.
13. 6.43875 ÷ 4027.5.
14. 9322.15 ÷ 6.275.
15. 7.2315 ÷ .0004751.
16. 480 ÷ 3.12.
17. 1.8 ÷ 28.8.
18. 17.1031 ÷ .53.
19. .00925 ÷ .37.
20. 3.72812 ÷ 4.07.

To divide a decimal by 10, 100, 1,000, &c., we move the decimal point as many places to the left as there are ciphers in the divisor, prefixing decimal ciphers, if necessary.

21. 347.2 ÷ 10. *Ans.* 34.72.
22. 817.4527 ÷ 100. *Ans.* 8.1745.
23. 17.4 ÷ 1,000. *Ans.* 0.0174.
24. 7852 ÷ 100. *Ans.* 78.52.
25. 3141 ÷ 10,000. *Ans.* .3141.

EXAMINATION QUESTIONS.

(83.) What is a power? (84.) What is a decimal fraction? a decimal? (85.) What is the method of writing a decimal? How are decimal places counted? (86) How many ways are there of reading decimals? Explain each. Repeat the extended numeration table. (87.) What are the fundamental principles of decimals? (89.) Give the rule for reducing decimals to common fractions. (90.) Give the rule for reducing common fractions to decimals. (91.) Define addition of decimals. (92.) Give the rule for adding decimals. (93.) Define subtraction of decimals. (94.) Give the rule for subtraction of decimals. (95.) What is multiplication of decimals? (96.) Give the rule for the multiplication of decimals. (97.) Define division of decimals. (98.) Give the rule for the division of decimals.

III. United States Currency.

Definition.

99. Currency is anything that circulates as money; as, *coin, government notes,* or *the notes of authorized banks.*

Decimal Currency.

100. The currency of the United States is reckoned in accordance with the decimal system, and for this reason it is called a *decimal currency.*

The primary unit of this currency is *one dollar;* the other units are derived from it in accordance with the following

TABLE.

10 mills (*m.*)	make	1 cent,	marked	*ct.*
10 cents	"	1 dime,	"	*d.*
10 dimes	"	1 dollar,	"	$
10 dollars	"	1 eagle,	"	*E.*

In practice the terms *eagle* and *dime* are but little used; the term *mill* is chiefly used in official reports and in laying taxes.

Notation.

101. Dollars and cents are usually written in the form of a mixed decimal, the decimal point being placed after the number of dollars. In this case, *eagles* occupy the place of *tens*, *dimes* the place of *tenths*, *cents* the place of *hundredths*, and *mills* the place of *thousandths*. Thus, 7 *eagles*, 4 *dollars*, 8 *dimes*, 5 *cents*, and 3 *mills*, is written $74.853, and is read 74 *dollars*, 85 *cents*, and 3 *tenths*.

EXAMPLES.

1. Write 34 *eagles*, 8 *dollars*, 4 *dimes*, and 7 *cents*.

Ans. $348.47.

2. Write 103 *eagles*, 2 *dollars*, 7 *dimes*, and 4 *cents*.
Ans. $1,032.74.

3. Write 3 *eagles*, 0 *dollars*, 4 *dimes*, 2 *cents*, and 3 *mills*.
Ans. $30.423.

Expressions for United States currency are usually read as *dollars* and *cents*, or as *dollars*, *cents*, and *mills*.

Read the following:

4. $87.375. *Ans.* 87 *dollars*, 37 *cents*, and 5 *mills*.
5. $2.37½. *Ans.* 2 *dollars*, 37½ *cents*.
6. $13.625. *Ans.* 13 *dollars*, 62½ *cents*.
7. $11.637. *Ans.* 11 *dollars*, 63 *cents*, and 7 *tenths*.

Application of the Rules for Decimals to U. S. Currency.

102. Expressions for United States currency conform in all respects to the decimal system; they may therefore be operated on by the rules for operating on decimals.

EXAMPLES.

1. Add $81.053, $67.412, $93.172, and $14.38.
Ans. $256.017.

2. Add $59.317, $69.565, $8.213, and $7.775.
Ans. $144.87.

3. A. bought 16 hams for $31.87½, a bag of coffee for $17.92, a chest of tea for $12.75, and a firkin of butter for $21.37½; what did they all cost? *Ans.* $83.92.

4. B. bought a house for $5,000, a store for $6,290, merchandise for $23,654.12, a horse and wagon for $278.53, a farm for $9,371.60, bank stock to the amount of $11,500, and a watch for $92.72½; what did the whole cost him? *Ans.* $56,186.97½.

5. What is the sum of $861.55, $378.25, $461.37, $683.57, $728.39, and $477.08? *Ans.* $3590.21.

6. From $875.043 subtract $704.91. *Ans.* $\$170.13\frac{3}{10}$.

7. How much must be added to $617.37½ to make up the sum of $922.75? *Ans.* $305.37½.

8. A.'s income is $6,250 per year, of which he spends $3,142.75 and lays up the rest; what does he lay up?
Ans. $3,107.25.

9. What is the difference between $969.801 and $36.999? *Ans.* $932.802.

10. From the sum of $981.43 and $456.81 subtract $498.75. *Ans.* $939.49.

11. From $10,000 subtract the sum of $4,367.18 and $3,587.47. *Ans.* $2,045.35.

12. From the sum of $965 and $341.60 take the sum of $433.33 and $89.47. *Ans.* $783.80.

13. A man received the following sums: $27.40, $68.75, $810.47, $386.59, and $2.20; he paid out the following sums: $78.67, $129.72, $119.46, and $3.88; how much had he left *Ans.* $963.68.

14. A merchant at the beginning of the year had a stock of goods worth $10,500; during the year he bought new goods to the amount of $9,345.75, and sold to the amount of $13,450.95; at the close of the year he had

on hand goods worth $11,122.37; how much did he make during the year? *Ans.* $4,727.57.

To find the cost of any number of things we multiply the cost of one thing by the number of things.

15. What will 17 barrels of flour cost at $6.37½ per barrel? *Ans.* $108.37½.

16. What will 85½ pounds of tea cost at $1.37½ a pound? *Ans.* $117.56¼.

17. What is the cost of 311 yards of linen at 64½ cents per yard? *Ans.* $200.59½.

18. What will a cask of wine containing 41½ gallons come to at $3.12½ per gallon? *Ans.* $129.68¾.

19. What will 278 cords of wood come to at $9.62½ per cord? *Ans.* $2,675.75.

20. A dealer bought 291 barrels of flour at $7.20 a barrel, 2,961 bushels of corn at 54 *cents* a bushel, and 398 bushels of rye at 78 *cents* a bushel; how much did the whole cost? *Ans.* $4,004.58.

21. If it costs $3.37½ per week to keep a horse, what will it cost to keep 13 horses for 39½ weeks? *Ans.* $1,733.06¼.

22. If 17 *lbs.* of tea cost $12.375, what will 1 *lb.* cost? *Ans.* $0.7279.

23. If 20 bushels of rye cost $18.542, what will 1 bushel cost? *Ans.* $0.9271.

24. If a man can earn $500 in 13.125 weeks, how much can he earn in 1 week? *Ans.* $38.0952.

25. If a barrel of flour costs $5.75, how many barrels can be bought for $1,035? *Ans.* 180 *bbls.*

26. If 1,000 acres of land cost $17,586.1045, what does one acre cost? *Ans.* $17.5861.

27. If one acre costs $500, how many acres can be bought for $3,785.455? *Ans.* 7.5709 acres.

28. If 75.0325 cords of wood cost $600, what will 1 cord cost? *Ans.* $7.9965.

Use of Aliquot Parts.

103. An **aliquot part** of a number is one of its exact divisors, the quotient being an integral number.

An aliquot part of a number may be either a whole number or a mixed number.

The principal aliquot parts of a dollar are shown in the following

TABLE.

50 *cts.*,	equal	to	$\frac{1}{2}$	of	$1.	$12\frac{1}{2}$ *cts.*,	equal	to	$\frac{1}{8}$	of	$1.
$33\frac{1}{3}$ *cts.*,	"	"	$\frac{1}{3}$	"	"	10 *cts.*,	"	"	$\frac{1}{10}$	"	"
25 *cts.*,	"	"	$\frac{1}{4}$	"	"	$6\frac{1}{4}$ *cts.*,	"	"	$\frac{1}{16}$	"	"
20 *cts.*,	"	"	$\frac{1}{5}$	"	"	5 *cts.*,	"	"	$\frac{1}{20}$	"	"

If one thing costs any aliquot part of a dollar we may find the cost of any number of things by the following

RULE.

Divide the number of things by the number of times the price of one thing is contained in $1: the quotient will be the required number of dollars.

EXAMPLES.

1. What is the cost of 64 bushels of oats at 50 cents a bushel? *Ans.* $\$\frac{64}{2} = \$32.$

2. What will 116 pounds of beef cost at 20 cents a pound? *Ans.* $\$\frac{116}{5} = \$23.20.$

3. What will 250 melons cost at 25 cents each?
Ans. \$62.50.

4. What will 144 pencils cost at $12\frac{1}{2}$ cents each?
Ans. \$18.

5. What will 75 oranges cost at 5 cents each?
Ans. \$3.75.

6. What will 69 yards of sheeting cost at $33\frac{1}{3}$ cents a yard? *Ans.* \$23.

7. What will 50 dozen marbles cost at $6\frac{1}{4}$ cents a dozen? *Ans.* $\$3.12\frac{1}{2}$.

8. What will 73 *lbs.* of sugar cost at $12\frac{1}{2}$ cents a pound? *Ans.* \$9.125.

9. What will 17 dozen of eggs cost at 25 cents a dozen? *Ans.* \$4.25.

10. What will 117 quarts of berries cost at 20 cents a quart? *Ans.* \$23.40.

11. What will 47 *lbs.* of coffee cost at $33\frac{1}{3}$ cents a pound? *Ans.* \$15.6667.

12. What will 145 *lbs.* of rice cost at $6\frac{1}{4}$ cents a pound? *Ans.* \$9.0625.

13. If 1 *lb.* of coffee costs $33\frac{1}{3}$ cents, how much will 87.3 *lbs.* cost? *Ans.* \$29.10.

14. If 1 *lb.* of sugar costs $12\frac{1}{2}$ cents, how much will 315 *lbs.* cost? *Ans.* \$39.375.

Miscellaneous Applications.

104. The following are some of the methods of applying previous principles:

1°. To find the cost of any number of things, knowing the price *per hundred*, or *per thousand.*

RULE.

Multiply the cost of 100, or 1000 things, by the number of things, and move the decimal point 2, or 3 places to the left, as the case may be.

EXAMPLES.

1. What will 460 oranges cost at $3.50 per hundred?

Ans. $$\frac{\$3.50 \times 460}{100} = \$16.10.$$

2. What is the cost of 1,796 *ft.* of boards, at $3.00 per 1,000 *ft.*

Ans. $$\frac{\$3 \times 1796}{1000} = \$5.388.$$

3. What is the cost of 47,555 bricks, at $7.50 per 1,000? *Ans.* $356.66¼.

4. What is the freight on 8,714 pounds, at 62½ cents per hundred? *Ans.* $54.46¼.

5. What is the cost of 83,750 *ft.* of flagging stone, at $60 per thousand feet? *Ans.* $5,025.

6. What is the cost of 763 *lbs.* of pork, at $12.50 per 100 *lbs.*? *Ans.* $95.37½.

7. What is the cost of 511 *lbs.* of beef, at $7 per hundred? *Ans.* $35.77.

8. What is the cost of 1,432 *lbs.* of pork, at \$8.25 per hundred? *Ans.* \$118.14.

9. What is the cost of 8,741 feet of spruce plank, at \$30 per thousand? *Ans.* \$262.23.

10. What is the cost of 4,875 *ft.* of roofing boards, at \$17 per thousand? *Ans.* \$82.87½.

2°. To find the cost of any number of pounds, knowing the cost per ton of 2,000 *lbs.*

RULE.

Multiply half the cost of a ton by the number of pounds, and move the decimal point three places to the left.

EXAMPLES.

11. What is the cost of 3,475 *lbs.* of plaster, at \$7.50 per ton? *Ans.* $\frac{\$3.75 \times 3475}{1000} = \$13.03\frac{1}{8}$.

12. What is the cost of transportation of 6,742 *lbs.*, at \$7 per ton? *Ans.* \$23.597.

13. What is the cost of 6,527 *lbs.* of oats, at \$30.50 per ton? *Ans.* \$99.5368.

14. What is the cost of 18,747 *lbs.* of coal, at \$8.50 per ton? *Ans.* \$79.6748.

15. What is the cost of 8,142 *lbs.* of cast-iron, at \$100 per ton? *Ans.* \$407.10.

16. Find the value of 3,120 *lbs.* of wool, at \$660 per ton? *Ans.* \$1,029.60.

17. What is the cost of 1,620 *lbs.* of hay, at \$16 per ton? *Ans.* \$12.96.

18. What is the cost of 5,782 *lbs.* of pig iron, at \$22 per ton? *Ans.* \$63.602.

19. What is the cost of 7,711 *lbs.* of straw, at \$15 per ton? *Ans.* \$57.8325.

20. What is the cost of 8,824 *lbs.* of hay, at \$15 per ton? *Ans.* \$66.18.

3°. To find the cost of one thing when we know the cost of any number of things.

RULE.

Divide the entire cost by the number of things.

EXAMPLES.

21. If 27 bushels of potatoes cost \$10.12½, what is the cost of 1 bushel? *Ans.* $\frac{\$10.125}{27} = \$0.37\frac{1}{2}$.

22. Paid \$29.68 for 14 barrels of apples; what was that per barrel? *Ans.* \$2.12.

23. A grocer buys 3 barrels of eggs, each containing 108 dozen, for \$81; what do they cost per dozen? *Ans.* 25 cents.

24. If \$600 is divided equally between 24 persons, how much does each receive? *Ans.* \$25.

25. A. bought 17½ yards of silk for \$31.50; how much did he pay per yard? *Ans.* \$1.80.

26. If 29 barrels of flour cost $195.75, what does 1 barrel cost? *Ans.* $6.75.

27. If 139 barrels of beef cost $2,189.25, what is the price per barrel? *Ans.* $15.75.

4°. Knowing the cost of any number of things, and the cost of one thing, to find the number of things.

RULE.

Divide the entire cost by the cost of one thing.

EXAMPLES.

28. How many barrels of apples can be bought for $47.50, at the rate of $2.37½ a barrel? *Ans.* 20.

29. Bought a chest of tea for $6.75, at the rate of 75 cents a pound; how many pounds did it contain? *Ans.* 9.

30. If 1 acre of land cost $28.75, how many acres can be bought for $3,220? *Ans.* 112.

31. If land costs $55 per acre, how much can be bought for $3,960? *Ans.* 72 *acres.*

32. If butter costs 22 cents a pound, how much can be bought for $19.36? *Ans.* 88 *lbs.*

33. If coal costs $8 per ton, how much can be bought for $108? *Ans.* 13½ *tons.*

34. Bought 115 dozen of eggs for $25.30; what did they cost per dozen? *Ans.* 22 *cts.*

35. Bought a piece of muslin containing 42¼ yards for $10.14; what did it cost per yard? *Ans.* 24 *cts.*

5°. To find the amount of a bill of items.

RULE.

Find the amount of each item separately, and take the sum of the results.

The sign @ stands for *at*.

EXAMPLES.

Find the amount of the following bills:

(36.)

MR. G. S. TOMPKINS,

Bought of J. MADDEN & Co.

16	pounds of tea	@	$0.85	$13	60
27	" coffee	@	.25	6	75
17	gallons of molasses	@	.92	15	64
112	pounds of sugar	@	.14	15	68
			Total,	$51	67

(37.)

P. G. BISSEL,

Bought of L. SMITH.

2462	feet of hemlock boards . . .	@	$7 per 1000.	
5410	" " " " . . .	@	$10 per 1000.	
600	" " scantling	@	$11.75 per 1000.	
1012	" " plank	@	$1.25 per 100.	
77	" " hewn timber	@	15 cts. per foot.	
			Total,	$102.584

(38.)

S. L. MORSE,

To J. BRISTOW, *Dr.*

To labor, self, 4 days	@	$4		
" " man, 6 days	@	$3.		
" materials		$17.		
		Total,	$51	00

(39.)

D. PLANT,

To JOHN DUFFIE, *Dr.*

To 17 yds. of calico	@ 12½ cts.		
" 12 yds. of muslin	@ 17 cts.		
" 2½ yds. of linen	@ 73 cts.		
" 9 spools thread	@ 7 cts.		
	Total,	$6	62

(40.)

J. D. SMITH,

Bought of A. HINE.

32 yds. silk	@ $2.12½		
18 yds. alpaca	@ 87½ cts.		
16 yds. chintz	@ 24 cts.		
42 yds. muslin	@ 21 cts.		
15 pieces tape	@ 14 cts.		
3 pairs gloves	@ $1.94		
12 pairs stockings	@ 62½ cts.		
	Total,	$111	83

EXAMINATION QUESTIONS.

(99.) What is currency? Illustrate. **(100.)** What is decimal currency? Recite the U. S. currency table. **(101.)** How are dollars and cents written? **(103.)** What is an aliquot part of a number? Its use? **(104.)** Give rules for finding: 1°. The cost of a number of things when we know the cost of 100 or 1000; 2°. The cost of a number of pounds when we know the cost of 1 ton; 3°. The cost of one thing when we know the cost of any number of things; 4°. The number of things when we know the whole cost and the cost of one thing; 5°. The amount of a bill of items.

CHAPTER IV.

COMPOUND NUMBERS.

I. DEFINITIONS AND TABLES.

Definitions.

105. A **denominate number** is one whose unit is named; as, 3 *feet*, 5 *pounds*, (Art. **5**).

Numbers that have the same unit are of the same denomination; numbers that have different units are of different denominations. Thus, 3 *feet* and 7 *feet* are of the same denomination; 3 *feet* and 7 *yards* are of different denominations.

106. A **compound number** is a collection of units of different denominations; as, 3 *feet* 6 *inches*, 3 *pounds* 6 *ounces*, (Art. **5**).

The different parts of a compound number must have a common primary unit, that is, they must be of the same *species*. Thus, in the compound number 3 *weeks* 4 *days*, 1 *day* is a primary unit of 3 weeks, as well as of 4 days.

If two numbers are of the same species, that which has the *greater unit* is said to be of the *higher* denomination. Thus, 3 *feet* is of a higher denomination than 6 *inches*.

Scales of Compound Numbers.

107. The **scale** of a compound number is a succession of simple numbers showing the relation between its different units.

If the scale shows how many times each unit is contained in one of the next higher denomination, it is said to be *ascending;* if it shows how many times each unit contains one of the next lower denomination, it is said to be *descending.*

Thus, in English currency, 4 *farthings* make 1 *penny*, 12 *pence* make 1 *shilling*, and 20 *shillings* make 1 *pound;* hence, the *ascending scale* of English currency is

4, 12, 20,

and the *descending scale* is

20, 12, 4.

In the ascending scale 4 *connects* farthings and pence, 12 *connects* pence and shillings, and 20 *connects* shillings and pounds.

In the descending scale 20 *connects* pounds and shillings, 12 *connects* shillings and pence, and 4 *connects* pence and farthings.

In United States currency the ascending and descending scales are both the same, viz.:

10, 10, 10, 10.

The scale of English currency is *varying*, that of United States currency is *uniform.*

The scales of some of the most important compound numbers are indicated in the following *tables.*

TABLES OF CURRENCY.

1°. United States Currency.

108. The United States currency was established by act of Congress in 1792.

TABLE.

10 mills (*m.*) make 1 cent . . . . . . *ct.*
10 cents " 1 dime . . . . . . *d.*
10 dimes " 1 dollar . . . . . $.
10 dollars " 1 eagle . . . . . *E.*

The primary unit of this currency is $1.

Canada Currency.

The currency of the Dominion of Canada is decimal, and, like that of the United States, it is reckoned in *dollars* and *cents.*

2°. English Currency.

109. This is the national currency of Great Britain:

TABLE.

4 farthings (*far.* or *qr.*) make 1 penny . . . *d.*
12 pence " 1 shilling . . *s.*
20 shillings " 1 pound . . . £.
21 shillings " 1 guinea. . . *G.*

EQUIVALENTS.

$$\pounds 1 = 20\,s. = 240\,d. = 960\,qrs.$$
$$1\,s. = 12\,d. = 48\,qrs.$$
$$1\,d. = 4\,qrs.$$

The primary unit of this currency is 1 pound sterling, and the corresponding coin is called a *sovereign*. The value of the sovereign is $4.8665.

The symbol £ is usually written before the number to which it refers.

3°. French Currency.

110. This is the national currency of France:

TABLE.

10 centimes (*cent.*) make 1 decime . . . . *d.*
10 decimes " 1 franc . . . . . *fr.*

The primary unit of this currency is 1 franc. Its value is 19.3 *cents*, that is, $1 is equal to about 5.18 francs.

In business transactions, French money is usually expressed in francs and decimals of a franc.

TABLES OF WEIGHT.

1°. Troy Weight.

111. This is used in weighing *gold*, *silver*, and some kinds of precious stones.

TABLE.

24 grains (*gr.*) make 1 pennyweight . *dwt.*
20 pennyweights " 1 ounce . . . . . . *oz.*
12 ounces " 1 pound . . . . . *lb.*

EQUIVALENTS.

1 *lb.* = 12 *oz.* = 240 *dwts.* = 5760 *grs.*
1 *oz.* = 20 *dwts.* = 480 *grs.*
1 *dwt.* = 24 *grs.*

The *pound Troy* is the primary unit of weight in Great Britain and the United States. It was declared by an act of Parliament, which took effect in 1826, that *the brass weight of one pound Troy made in the year* 1758, which was then in the custody of the Clerk of the House of Commons should be the *unit*, or *standard weight*, from which all other weights should be derived. It was also enacted that the pound Troy should contain 5,760 *grains*, and that 7,000 such grains should make a *pound avoirdupois.*

The English system of weights and measures has been adopted by act of Congress, and is the legal system of the United States.

2°. Apothecaries' Weight.

112. This differs from Troy weight in the mode of subdividing the ounce; it is used in weighing medicines and the like.

TABLE.

20 grains (*gr.*)	make	1 scruple . . . . ℈
3 scruples	"	1 dram . . . . . ʒ
8 drams	"	1 ounce . . . . . ℥
12 ounces	"	1 pound . . . . . ℔

EQUIVALENTS.

1 ℔ = 12 ℥ = 96 ʒ = 288 ℈ = 5760 *grs.*
1 ℥ = 8 ʒ = 24 ℈ = 480 *grs.*
1 ʒ = 3 ℈ = 60 *grs.*
1 ℈ = 20 *grs.*

3°. Avoirdupois Weight.

113. This weight is used in weighing heavy articles, as hay, flour, salt, &c., and all metals, except gold and silver.

TABLE.

16 ounces (*oz.*)	make	1 pound	*lb.*
25 pounds	"	1 quarter	*qr.*
4 quarters	"	1 hundredweight	*cwt.*
20 hundredweight	"	1 ton	*T.*

EQUIVALENTS.

1 *T.* = 20 *cwt.* = 80 *qrs.* = 2000 *lbs.* = 32000 *oz.*
1 *cwt.* = 4 *qrs.* = 100 *lbs.* = 1600 *oz.*
1 *qr.* = 25 *lbs.* = 400 *oz.*
1 *lb.* = 16 *oz.*

The primary unit of this system is 1 *pound*, equal to 7,000 grains Troy.

In weighing coarse and heavy articles, as coal at the mines, and the like, it is customary to call 112 *lbs.* a *hundredweight*, and 28 *lbs.* a *quarter*.

MEASURES OF TIME.

114. Time is a measured portion of duration.

The period required for the earth to revolve once on its axis is called a *day*. The time required for the earth to revolve around the sun is called a *year*. These are natural units of time.

Civil Time.

115. The astronomical year does not contain an exact number of days; hence, for civil purposes an artificial year is adopted. Sometimes the civil year contains 365 days and sometimes it contains 366 days. Years containing 365 days are called *common years* and those containing 366 days are called *leap years*. These are so distributed that for long periods the *average* length of the civil year is equal to that of the astronomical year.

Every year divisible by 4 (except centennial years) is a leap year, and all the rest are common years; thus, 1876 is a leap year, but 1875 and 1877 are common years.

Every centennial year that is divisible by 400 is a leap year, and all the rest are common years; thus, the year 2000 is a leap year, but 1800 and 1900 are common years.

TABLE.

60 seconds (*sec.*)	make	1 minute	*min.*
60 minutes	"	1 hour	*hr.*
24 hours	"	1 day	*da.*
7 days	"	1 week	*wk.*
365 days	"	1 common year . .	*yr.*
366 days	"	1 leap year	*yr.*
100 years	"	1 century.	*C.*

EQUIVALENTS.

1 *wk.* = 7 *da.* = 168 *hrs.* = 10080 *min.* = 604800 *sec.*
1 *da.* = 24 *hrs.* = 1440 *min.* = 86400 *sec.*
1 *hr.* = 60 *min.* = 3600 *sec.*
1 *min.* = 60 *sec.*

The civil year is divided into 12 parts called *months.* The order of the months and the number of days in each is shown in the following

TABLE.

1°. January	31 days.	7°. July	31 days.
2°. February	28 or 29 "	8°. August	31 "
3°. March	31 "	9°. September	30 "
4°. April	30 "	10°. October	31 "
5°. May	31 "	11°. November	30 "
6°. June	30 "	12°. December	31 "

In common years February has 28 days; in leap years it has 29.

GEOMETRICAL MAGNITUDES.

Definitions.

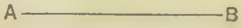
116. A **magnitude** is anything that can be measured.

To measure a thing, is to find how many times it contains a thing of the same kind, taken as a unit.

117. There are four kinds of geometrical magnitudes, viz.: *lines*, *surfaces*, *volumes*, and *angles.*

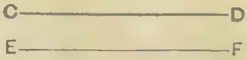
118. A **line** is a magnitude that has length, without breadth or thickness.

A **straight line** is one whose direction does not change at any point; as AB.



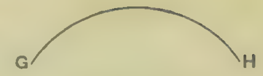
STRAIGHT LINE.

Straight lines are *parallel* when they have the same direction; as, CD and EF.



PARALLEL LINES.

A **curved line** is one whose direction changes at every point; as, GH.



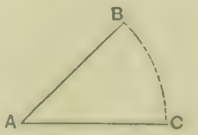
CURVED LINE.

119. A **surface** is a magnitude that has length and breadth, without thickness.

A **plane** is a surface such that if a straight line is applied to it in any direction it will coincide with the surface throughout.

120. A **volume** is a magnitude that has *length, breadth,* and *thickness* or *height.*

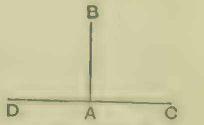
121. An **angle** is the opening between two lines that meet at a point; as, BAC.



ANGLE

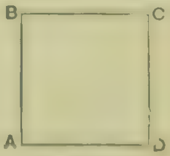
The lines AB and AC are called *sides,* and the point A is called the *vertex* of the angle.

If one straight line meets another so as to make the two adjacent angles equal, the first line is said to be perpendicular to the second, and the angles are called *right angles;* thus, if the angles BAD and BAC are equal they are both right angles and BA is perpendicular to DC.



RIGHT ANGLES.

122. A **square** is a portion of a plane bounded by four equal lines, called *sides,* and having all its angles right angles; as, ABCD.

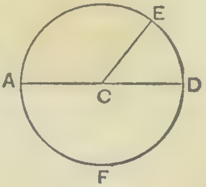


SQUARE.

123. A **circle** is a portion of a plane bounded by a curved line every point of which is equally distant from a point within called the centre. The bounding line is called the *circumference*, and any part of this is called an *arc* of the circle.

A **diameter** is a line passing through the centre and terminating in the circumference.

A **radius** is a line drawn from the centre to any point of the circumference.

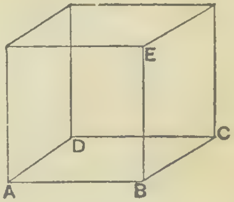


CIRCLE.

Thus, AEDF is a circle, C its centre, AD a diameter, and CE a radius.

124. A **cube** is a volume bounded by six equal squares, called *faces;* the sides of the squares are called *edges* of the cube.

Thus, ABCD–E is a cube, ABCD is one of its faces, AB, BC, and BE are edges.



CUBE.

MEASURES OF LENGTH.

1°. Long Measure.

125. This measure is used for measuring distances, and the dimensions of objects.

TABLE.

12	inches (*in.*)	make	1 foot	. . . *ft.*
3	feet	"	1 yard	. . . *yd.*
5½	yards	"	1 rod	. . . . *rd.*
40	rods	"	1 furlong	. *fur.*
8	furlongs, or 320 rods	"	1 mile	. . . *mi.*
3	miles	"	1 league	. . *lea.*

EQUIVALENTS.

$$\begin{aligned} 1\,mi. = 320\,rds. &= 5280\ ft. = 63360\,in. \\ 1\,rd. &= 16\tfrac{1}{2}\,ft. = 198\,in. \\ &\ \ 1\,ft. = 12\,in. \end{aligned}$$

It is found convenient in practice to reduce *yards, feet,* and *inches* to *half yards* and *inches;* we thus avoid the inconvenience of a scale in which one of the numbers is fractional. In this case, the first part of the preceding table may be replaced by the following

TABLE.

18 inches make 1 half yard . . . . *hf. yd.*
11 half yards " 1 rod . . . . *rd.*

The *yard* is the primary unit of English and American measures of length. By the act of Parliament already referred to, it was declared that the brass standard yard made in 1760, then in the custody of the Clerk of the House of Commons, should be the *imperial standard yard.* From it we derive all other measures of lines, surfaces, and volumes.

2°. Surveyors' Measure.

126. This is used in measuring land. The unit is a Gunter's chain; this chain is 4 rods, or 66 feet, in length and is divided into 100 equal parts, called *links.*

TABLE.

7.92 inches make 1 link . . . . . . *li.*
100 links " 1 chain . . . . . *ch.*
80 chains " 1 mile . . . . . *mi.*

3°. Cloth Measure.

127. This is used in measuring cloth, ribbons, and the like. Its primary unit is 1 *yard*.

TABLE.

2¼	inches (*in.*)	make	1 nail.	*na.*
4	nails	"	1 quarter. . .	*qr.*
4	quarters	"	1 yard	*yd.*

EQUIVALENTS.

$$
\begin{aligned}
1\,yd. = 4\,qrs. &= 36\,in. \\
1\,qr. &= 9\,in.
\end{aligned}
$$

In business transactions the yard is usually subdivided into *halves*, *quarters*, *eighths*, and *sixteenths*. For custom-house purposes it is divided into *tenths*.

MEASURES OF SURFACE.

128. A **unit of surface** is a square whose sides are equal to some unit of length. If each side is a lineal yard, the square is called a *square yard*.

The **area of a surface** is an expression for that surface in terms of a square unit.

1°. Square Measure.

129. This is used in measuring all kinds of surfaces:

TABLE.

144 square inches (*sq. in.*) make 1 square foot . . *sq. ft.*
9 square feet " 1 square yard. . *sq. yd.*
30¼ square yards " 1 square rod . . *sq. rd.*
160 square rods " 1 acre . . . . . . *A.*

EQUIVALENTS.

$$\begin{aligned} 1\,A. = 160\,\textit{sq. rds.} = 4840\ \textit{sq. yds.} &= 43560\ \textit{sq. ft.} = 6272640\,\textit{sq. in.} \\ 1\,\textit{sq. rd.} = 30\tfrac{1}{4}\,\textit{sq. yds.} &= 272\tfrac{1}{4}\,\textit{sq. ft.} = 39204\,\textit{sq. in.} \\ 1\ \textit{sq. yd.} &= 9\ \textit{sq. ft.} = 1296\,\textit{sq. in.} \\ 1\ \textit{sq. ft.} &= 144\,\textit{sq. in.} \end{aligned}$$

It is found convenient in practice to reduce *square yards*, *square feet*, and *square inches*, to *quarter square yards* and *square inches*; we thus avoid using a scale containing a fractional number. In this case we have the following

TABLE.

324 square inches make 1 quarter square yard . *qr. sq. yd.*
121 square quarters " 1 square rod . . . . *sq. rd.*
160 square rods " 1 acre . . . . *A.*

2°. Land Measure.

130. This is used in measuring land:

TABLE.

10000 square links (*sq. li.*) make 1 square chain . . *sq. ch.*
10 square chains " 1 acre . . . . *A.*
640 acres, " 1 square mile . . *sq. mi.*

In government surveys, a square mile is called a *section*; 36 sections make 1 township.

MEASURES OF VOLUME AND CAPACITY.

131. A unit of volume is a cube whose edges are equal to some unit of length. If each edge is a lineal yard, the cube is called a *cubic yard.*

The contents of a volume is an expression for that volume in terms of a cubic unit.

A **unit of capacity** is a measure having a determinate content, or capacity.

1°. Cubic Measure.

132. This is used in measuring all kinds of volumes:

TABLE.

1728 cubic inches (*cu. in.*) make 1 cubic foot . . . *cu. ft.*
27 cubic feet " 1 cubic yard . . *cu. yd.*
128 cubic feet " 1 cord . . . . . . *C.*

A *cord of wood* is a pile 4 feet wide, 4 feet high, and 8 feet long. A foot in length of such a pile is a *cord foot;* 8 *cord feet* make 1 *cord.*

2°. Dry Measure.

133. This is used in measuring grain, fruit, salt, and the like.

TABLE.

2 pints (*pt.*) make 1 quart. . . . . . *qt.*
8 quarts " 1 peck . . . . . . *pk.*
4 pecks " 1 bushel . . . . . *bu.*

EQUIVALENTS.

$$\begin{aligned}1\,bu. = 4\,pks. &= 32\,qts. = 64\,pts.\\ 1\,pk. &= 8\,qts. = 16\,pts.\\ 1\,qt. &= 2\,pts.\end{aligned}$$

The primary unit is the *bushel*. The bushel (known as the Winchester bushel) is a cylindrical measure $18\frac{1}{2}$ inches across and 8 inches deep; it contains $2{,}150\frac{2}{5}$ cubic inches *nearly*.

3°. Liquid Measure.

134. This is used in measuring liquids:

TABLE.

4	gills (*gi.*)	make	1 pint	*pt.*
2	pints	"	1 quart	*qt.*
4	quarts	"	1 gallon	*gal.*
$31\frac{1}{2}$	gallons	"	1 barrel	*bbl.*
2	barrels	"	1 hogshead	*hhd.*
2	hogsheads	"	1 pipe	*pi.*
2	pipes	"	1 tun	*tun.*

EQUIVALENTS.

$$\begin{aligned}1\,bbl. = 31\tfrac{1}{2}\,gals. &= 126\,qts. = 252\,pts. = 1008\,gi.\\ 1\,gal. &= 4\,qts. = 8\,pts. = 32\,gi.\\ 1\,qt. &= 2\,pts. = 8\,gi.\\ 1\,pt. &= 4\,gi.\end{aligned}$$

The primary unit is 1 *gallon*; it contains 231 cubic inches. The liquid quart is about $\frac{6}{7}$ of a quart of dry measure.

ANGULAR MEASURE.

135. In measuring angles, the right angle is taken as the primary unit. The ninetieth part of a right angle is called a *degree.*

TABLE.

60 seconds (″) make 1 minute . . . . ′
60 minutes " 1 degree. . . . . °
90 degrees " 1 right angle . . *rt. a.*

EQUIVALENTS.

$$\begin{aligned} 1^\circ &= 60' = 3600'' \\ 1' &= 60'' \end{aligned}$$

If we divide the circumference of a circle into 360 equal arcs, (Art. **123**), each arc is called a degree, because the angle contained between the radii drawn to the extremities of such an arc is equal to 1 degree. Hence, we say,

360 degrees make 1 circumference . . . *cir.*

MISCELLANEOUS TABLES.

136. The following miscellaneous tables are often used in operating on compound numbers:

1°. Counting.

12 things make 1 dozen . . . . . . *doz.*
12 dozen " 1 gross. . . . . . . *gr.*
12 gross " 1 great gross . . . *g. gr.*

2°. Paper.

24 sheets make 1 quire. . . . . . . *qr.*
20 quires " 1 ream. . . . . . . *ream.*
2 reams " 1 bundle. . . . . . *bund.*
2 bundles " 1 bale.

3°. Books.

A book in which
each sheet is folded into 2 leaves is a folio.
" " " " 4 " " quarto, or 4to.
" " " " 8 " " octavo, or 8vo.
" " " " 12 " " duodecimo, or 12mo.
" " " " 16 " " 16mo.
" " " " 24 " " 24mo.
" " " " 32 " " 32mo.

4°. Miscellaneous Values.

4 *inches* make 1 hand, used in measuring horses.
9 " " 1 span, used in sacred history.
18 " " 1 cubit, " " " "
6 *feet* " 1 fathom, used in measuring depths.

THE METRIC SYSTEM.

137. The metric system is a system of weights and measures based on a primary unit of length called a *meter*.

This system was first adopted by France and afterwards by various other countries. Since 1866 its use has been permitted in the United States, by act of Congress.

The *meter* is approximately equal to $\frac{1}{10000000}$ of the distance from the equator to the north pole, measured on the meridian through Paris. It is nearly equal to 39.37 inches.

The scales of all compound numbers in the metric system are decimal.

The names of units in the ascending scale are formed by prefixing the following numerals to the names of the primary units: *deca*, ten; *hecto*, one hundred; *kilo*, one thousand; and *myria*, ten thousand.

The names of units in the descending scale are formed by prefixing the following numerals to the names of the primary units: *deci*, one tenth; *centi*, one hundredth; and *milli*, one thousandth.

Measures of Length.

138. The primary unit is the *meter*.

TABLE.

10 millimeters (*mm.*)	make	1 centimeter	=	.3937	*in.*
10 centimeters	“	1 decimeter	=	3.937	“
10 decimeters	“	1 meter (*m.*)	=	39.37	“
10 meters	“	1 decameter	=	393.7	“
10 decameters	“	1 hectometer	=	328 *ft.* 1	*in.*
10 hectometers	“	1 kilometer	=	3280 *ft.* 10	*in.*
10 kilometers	“	1 myriameter	=	6.2137	*mi.*

Lengths are usually expressed decimally in terms of some one of the units of the preceding scale. Small distances are expressed in *millimeters*, ordinary distances in *meters*, and long distances in *kilometers*.

Measures of Surface.

139. The primary unit of ordinary surfaces is a *square meter*.

TABLE.

100 sq. centimeters make 1 sq. decimeter = 15.5 *sq. in.*
100 sq. decimeters " 1 sq. meter = 1550 " "

For land measure the primary unit is the *are* (pron. *air*); it is a square decameter, or 100 square meters.

100 sq. meters make 1 are = 119.6 *sq. yds.*
100 ares " 1 hectare = 2.471 *acres.*

Measures of Volume and Capacity.

140. The primary unit of ordinary volumes is the *stere* (pron. *stair*); it is a cubic meter.

TABLE.

1000 cu. centimeters make 1 cu. decimeter = 61.026 *cu. in.*
1000 cu. decimeters " 1 cu. meter = 35.316 *cu. ft.*

For wood measure the primary unit is the *stere* (pron. *stair*); it is a cubic meter.

10 decisteres make 1 stere = .2759 *cords.*
10 steres " 1 decastere = 2.759 "

For measures of capacity the primary unit is the *liter* (pron. *leeter*). It is a cubic decimeter.

TABLE.

			Dry Measure.		Liquid Measure.
10 centiliters	make	1 deciliter	= .0908 *qt.*	=	.1057 *qt.*
10 deciliters	“	1 liter	= .908 *qt.*	=	1.0567 *qt.*
10 liters	“	1 decaliter	= 9.08 *qts.*	=	2.6417 *gals.*
10 decaliters	“	1 hectoliter	= 2.8375 *bu.*	=	26.417 *gals.*

Measures of Weight.

141. The primary unit of weight is the *gramme*. It is the weight of a cubic centimeter of distilled water at 39° Fah.

TABLE.

10 milligrammes (*mg.*)	make	1 centigramme	=	.1543 *grs.*
10 centigrammes	“	1 decigramme	=	1.5432 “
10 decigrammes	“	1 gramme	=	15.432 “
10 grammes	“	1 decagramme	=	.3527 *oz. av.*
10 decagrammes	“	1 hectogramme	=	3.5274 “
10 hectogrammes	“	1 kilogramme	=	2.2046 *lbs. av.*

Small weights are expressed in *milligrammes* and large ones in *kilogrammes*. The kilogramme is the weight of a liter of water at 39° Fah.

A *quintal* is 100 kilogrammes or 220.46 *lbs. av.* The French *tonneau*, or metric ton, is the weight of a cubic meter of water, or to 2,204.6 *lbs. av.*

We may read a number in the metric system in terms of all its units, or in terms of any one of them. Thus, the expression 27.34 *meters* may be read 2 *decameters*, 7 *meters*, 3 *decimeters*, and 4 *centimeters*, or it may be read as it is written, 27 *meters and* 34 *hundredths of a meter*

EXAMINATION QUESTIONS.

(105.) What is a denominate number? When are numbers of the same denomination? When of different denominations? (106.) What is a compound number? Character of the parts? (107.) What is a scale? When ascending and when descending? Give the scale of English currency. Of United States currency. (114.) What is time? Civil time? How are civil years arranged? (116.) Define a magnitude? (118.) What is a line? A straight line? Parallels? A curved line? (119.) A surface? A plane? (120.) A volume? (121.) An angle? A right angle? (122.) Define a square. (123.) A circle. A diameter? A radius? (124.) What is a cube? (128.) What is a unit of surface? An area? (131.) What is a unit of volume? Contents? (137.) What is the metric system? A meter? How are the names of metric numbers formed?

II. REDUCTION.

142. Reduction is the operation of changing a number from one denomination to another, without altering its value.

Reduction may be *descending*, or *ascending*:

1°. Reduction descending is the operation of changing from a higher to a lower denomination.

2°. Reduction ascending is the operation of changing from a lower to a higher denomination.

Reduction Descending.

143. Let it be required to reduce £7 4s. 3*d*. to pence.

Explanation.—The given number contains 7 units, each equal to £1, and from the scale, (Art. 109), we see that each pound contains 20*s.*; multiplying 7 by 20 and adding 4, we have 144, that is, £7 4*s.* is equal to 144*s.*; but from the scale we see that each shillling is equal to 12*d.*; multiplying 144 by 12 and adding 3 we have 1731, that is, the given number is equal to 1731*d.*

OPERATION.

£7 4*s.* 3*d.*
20
144*s.*
12
1731*d.* *Ans.*

Since all similar cases may be treated in the same manner, we have the following

RULE.

I. Multiply the units of the highest denomination by the number of the scale that connects this denomination with the one next lower and add the units of the latter denomination to the product.

II. Multiply this result by the number that connects it with the next lower denomination and add the units of that denomination to the product.

III. Continue this operation till the required denomination is reached.

EXAMPLES.

1. How many farthings are there in £4 11*s.* 3*d.* 2 *far.*?
Ans. 4,382 *far.*

2. How many grains in 3 *lbs.* 8 *oz.* 6 *dwts.* 4 *grs.*?
Ans. 21,268 *grs.*

3. How many minutes in 3 *wks.* 5 *da.* 5 *hrs.*?
Ans. 37,740 *min.*

4. How many yards in 3 *mi.* 28 *rds.* 2 yds.?

Ans. 5,436 *yds.*

5. How many square feet in 40 square rods?

Ans. 10,890 *sq. ft.*

6. How many pints in 8 *bu.* 3 *pks.* 2 *qts.* 1 *pt.*?

Ans. 565 *pts.*

7. How many seconds in 4 *wks.* 3 *da.* 17 *hrs.*?

Ans. 2,739,600 *sec.*

8. In £31 8*s.* $9\frac{3}{4}$*d.*, how many farthings?

Ans. 30,183 *far.*

9. How many inches in 6 *rds.* 4 *yds.* 2 *ft.* 9 *in.*?

Ans. 1,365 *in.*

10. In 17.5 square chains, how many square links?

Ans. 175,000 *sq. li.*

11. In 25 cords of wood, how many cord feet and how many cubic feet? *Ans.* 200 *c. ft.* and 3,200 *cu. ft.*

12. How many quarts are there in 20 *bu.* 3 *pks.*?

Ans. 664 *qts.*

To reduce a *denominate fraction* to integral units of a lower denomination:

RULE.

Multiply the fraction by the number that connects it with the next lower denomination; then multiply the fractional part of the result by the number that connects it with the next lower denomination, and so on.

13. Reduce £$\frac{5}{6}$ to shillings and pence.

Multiplying by 20, we have £$\frac{5}{6}$ = $\frac{100}{6}$s. = $16\frac{2}{3}$s.;

Multiplying $\frac{2}{3}$s. by 12, we find $\frac{2}{3}$s. = 8*d.*;

Hence, £$\frac{5}{6}$ = $16\frac{2}{3}$s. = 16*s.* 8*d.* *Ans.*

14. Reduce £.875 to shillings and pence.

Multiplying by 20, we find £.875 = 17.5*s.*;
Multiplying .5*s.* by 12, we find .5*s.* = 6*d.*;
Hence, £.875 = 17*s.* 6*d.* *Ans.*

15. Reduce .9375 *lbs.* Troy to ounces and pennyweights. *Ans.* 11 *oz.* 5 *dwts.*

16. Reduce .11875 *tons* to lower denominations.
Ans. 2 *cwt.* 1 *qr.* 12 *lbs.* 8 *oz.*

17. Reduce 450 *sq. yds.* to square inches.

18. Reduce £$2\frac{3}{8}$ to farthings.

19. Reduce $3\frac{49}{96}$ *days* to seconds.

20. Reduce $141\frac{3}{4}$ *weeks* to minutes.

21. Reduce $4\frac{11}{20}$ *tons* to ounces.

22. Reduce $3\frac{12903}{16000}$ *tons* to ounces.

23. Reduce $2\frac{81}{176}$ *miles* to feet.

24. Reduce $7\frac{347}{660}$ *furlongs* to inches.

25. Reduce $3\frac{19}{32}$ *bushels* to pints.

26. Reduce $43\frac{3}{16}$ *yards* to inches.

27. Reduce $17\frac{3}{8}$ *yards* to inches.

28. Reduce $\frac{4}{17}$ of a hogshead to units of a lower denomination.

29. Reduce $\frac{5}{27}$ of a degree to units of a lower denomination.

30. Reduce $\frac{4}{11}$ of a ton to units of a lower denomination.

31. Reduce $\frac{5}{7}$ of a week to units of a lower denomination.

32. Reduce £$\frac{23}{27}$ to units of a lower denomination.

33. Reduce $\frac{7}{9}$ *bu.* to units of a lower denomination.

34. Reduce $2\frac{1}{2}$ *T.* to ounces.

Reduction Ascending.

144. Let it be required to reduce 1,731 *pence* to *pounds, shillings,* and *pence:*

Explanation.—We see from the scale, (Art. 109), that 12 connects pence with shillings; dividing 1,731 by 12, we find a quotient 144 with a remainder 3; hence the given number is equal to 144*s.* 3*d.*; in like manner dividing 144 by 20, we find a quotient 7 and a remainder 4, that is, 144*s.* is equal to £7 4*s.*; hence, the given number is equal to £7 4*s.* 3*d.*

OPERATION.

$$\begin{array}{l} 12\underline{)1731d.} \\ \;\;20\underline{)144s.} \ldots 3d. \\ \;\;\;\;\;\;\text{£}7 \;\ldots 4s. \end{array}$$

£7 4*s.* 3*d.* *Ans.*

In the manner we may treat all similar cases; hence, the following

RULE.

I. Divide the units of the given denomination by the number of the scale that connects this denomination with the one next higher: the the remainder will be units of the same denomination as the dividend.

II. Divide the quotient by the number that connects it with the next higher denomination: the remainder will be units of the same denomination as the new dividend.

III. Continue the operation till the required denomination is reached.

EXAMPLES.

1. Reduce 15,732 grains to pounds Troy.

Ans. 2 *lbs.* 8 *oz.* 15 *dwts.* 12 *grs.*

2. Reduce 525,960 minutes to weeks.

Ans. 52 *wks.* 1 *da.* 6 *hrs.*

Proof.—Reduction descending is proved by reduction ascending, and the reverse; thus, the preceding example is proved by reducing 52 *wks.* 1 *da.* 6 *hrs.* to minutes; this gives 525,960 *min.*

Let each of the following examples be proved:

3. Express 49,180 *grains* in *pounds*, Apothecaries' weight.
Ans. 8℔. 6℥ 3ʒ 2℈.

4. Reduce 3,392 *grains* to *ounces*, Apothecaries' weight.
Ans. 7℥ 1℈ 12 *grs.*

5. Reduce 2,945 *inches* to *rods.*

In this case we reduce 2,945 inches to *half yards;* then we reduce the *half yards* to rods, (Art. 125). Dividing 2,945 *in.* by 18, because there are 18 *inches* in a half yard, we find 163 *half yards* and a remainder of 11 *inches;* dividing 163 by 11, because there are 11 *half yards* in a rod, we find 14 *rods* with a remainder equal to 9 *half yards*, or to 4 *yards* and 18 *inches;* adding the 18 *inches* to the first remainder, 11 *inches*, we find 29 *inches*, or 2 *ft.* 5 *in.* Hence,

2,945 *in.* = 14 *rds.* 4 *yds.* 2 *ft.* 5 *in.*

6. Reduce 1,365 *inches* to *rods.*
Ans. 6 *rds.* 4 *yds.* 2 *ft.* 9 *in.*

7. Reduce 272,668 *inches* to the highest possible denomination. *Ans.* 4 *mi.* 97 *rds.* 1 *ft.* 10 *in.*

8. Reduce 88,435 inches to its highest denomination.
Ans. 1 *mi.* 126 *rds.* 3 *yds.* 1 *ft.* 7 *in.*

9. Reduce 873 *oz.* avoirdupois to its highest denomination. *Ans.* 2 *qrs.* 4 *lbs.* 9 *oz.*

10. Reduce 7,634 gills to gallons, &c.
Ans. 238 *gals.* 2 *qts.* 2 *gi.*

To reduce a compound number to a fractional part of some unit of a higher denomination:

RULE.

Reduce the compound number to its lowest denomination; also reduce the given unit to the same denomination; then divide the former by the latter.

11. Reduce 7*s.* 6*d.* to a fraction of £1.

The given number is equal to 90*d.*, and £1 is equal to 240*d.*; hence 7*s.* 6*d.* = £$\frac{90}{240}$ = £$\frac{3}{8}$. *Ans.*

12. What part of a tun is 3 *hhds.* 31 *gals.* 2 *qts.*?

Ans. $\frac{7}{8}$.

13. What part of a hogshead is 3 *gals.* 2 *qts.*?

Ans. $\frac{1}{18}$.

14. What part of a mile is 116 *rds.* 2 *yds.*? *Ans.* $\frac{4}{11}$.

15. What part of a right angle is 3° 15′ 12″?

Ans. $\frac{122}{3375}$.

16. What part of a gallon is 2 *qts.* 1 *pt.* 1 *gi.*?

Ans. $\frac{21}{32}$.

To reduce a compound number to a decimal of a higher denomination:

RULE.

Divide the units of the lowest denomination by the number of the scale that connects this denomination with the next higher one, (Art. 90), and annex the quotient to the units of that denomination; then divide this result by the number that connects it with the next higher denomination and annex the quotient to the units of that denomination; and so on, till the required unit is reached.

17. Reduce 15 *cwt.* 3 *qrs.* 2½ *lbs.* to the decimal of a ton.

OPERATION.

25	2.5 *lbs.*
4	3.1 *qrs.*
20	15.775 *cwt.*
	.78875 *T.*

Explanation.—For convenience the units of the several denominations are written in a vertical column, the least denomination at the top, and the numbers of the ascending scale are written on the left from the top downward. Dividing 2½ *lbs.*, or its equal 2.5 *lbs.* by 25, to reduce it to quarters, we have .1 *qr.*, which being annexed to 3 *qrs.* gives 3.1 *qrs.*; dividing 3.1 *qrs.* by 4, to reduce it to hundreds, we have .775 *cwt.*, which being annexed to 15 *cwt.* gives 15.775 *cwt.*; dividing this by 20 to reduce it to tons, we have .78875 *T.*, which is the answer.

18. Reduce 3*s.* 6*d.* to the decimal of a pound sterling.
Ans. £.175.

19. Reduce £14 17*s.* 3*d.* to pounds and decimals.
Ans. £14.8625.

20. Reduce 7 *mi.* 281 *rds.* to miles. *Ans.* 7.8781 *mi.*

21. Reduce 2 *cwt.* 1 *qr.* 12 *lbs.* 8 *oz.* to tons.
Ans. .1188 *ton.*

22. Reduce 3¼ ℥ to a decimal of a pound.
Ans. .2708 ℔.

23. Reduce 2⅓ *pts.* to a decimal of 1 gallon.
Ans. .2917 *gal.*

24. Reduce 21 *inches* to a decimal of a yard.
Ans. .5833 *yd.*

A compound number may be expressed in terms of any of its denominations; thus, £3 4*s.* 9*d.* expressed in shillings is equal to 64.75*s.*

25. Reduce 2 *lbs.* 4 *oz.* 5 *dwts.* to ounces.
Ans. 28.25 *oz.*

26. Reduce 2 *T.* 4 *cwt.* 3 *qrs.* 5 *lbs.* to *cwts.*

Ans. 44.8 *cwts.*

27. Reduce 8 *bu.* 3 *pks.* 6 *qts.* to pecks.

Ans. 35.75 *pecks.*

28. Reduce £5 4*s.* 9*d.* to shillings.

29. Reduce 3 *bbls.* 14 *gals.* 3 *qts.* 1 *pt.* to gallons.

30. Reduce 12 *mi.* 47 *rds.* 4 *yds.* 2 *ft.* to rods.

31. Reduce 47° 35′ 42″ to minutes.

32. What part of £3 10*s.* 6*d.* is £1 15*s.* 4*d.*?

In examples like the last we reduce both numbers to a common unit and then perform the indicated operation. Thus, £3 10*s.* 6*d.* = 846*d.*, and £1 15*s.* 4*d.* = 424*d.*; hence the required answer is $\frac{424}{846}$, or $\frac{212}{423}$.

33. What part of a ton is 4 *cwt.* 23 *lbs.*?

34. What part of a week is 6 *da.* 12 *hrs.* 30 *min.*?

35. What part of a mile is 211 *rds.* 5 *yds.*?

36. What part of 36° 15′ 20″ is 11° 13′ 20″?

37. What part of 4 *gals.* 3 *qts.* is 2 *gals.* 3 *qts.*?

38. What part of 2 *T.* 6 *cwt.* 20 *lbs.* is 7 *cwt.* 10 *lbs.*?

39. What part of £$\frac{3}{8}$ is $1\frac{1}{4}$ *s.*?

40. What part of $\frac{7}{8}$ *mi.* is $4\frac{3}{4}$ *rds.*?

MISCELLANEOUS EXAMPLES IN REDUCTION.

1. How many grains in a tankard that weighs 1 *lb.* 11 *oz.* 15 *dwts.*? *Ans.* 11,400 *grs.*

2. In 97,397 grains of gold how many pounds?

Ans. 16 *lbs.* 10 *oz.* 18 *dwts.* 5 *grs.*

3. In 24 *tons* 17 *cwt.* 3 *qr.* how many pounds?

Ans. 49,775 *lbs.*

4. In $136\frac{1}{2}$ bushels how many quarts?

Ans. 4,368 *qts.*

5. How many miles in 1,571,328 inches?

Ans. 24.8 *mi.*

6. In 2,624 cubic feet of wood, how many cords?

Ans. $20\frac{1}{2}$ C.

7. Reduce 18,545,435 *sec.* to days.

Ans. 214 *d.* 15 *h.* 30 *m.* 35 *sec.*

8. What is the value of $\frac{3}{8}$ of 1*s.*? *Ans.* $4\frac{1}{2}$*d.*

Metric weights and measures may be converted into English weights and measures by the tables of equivalents given in Arts. **138–141.**

9. How many inches in 7 meters?

Ans. $39.37 \times 7 = 275.59$.

10. How many meters in 605 inches?

Ans. $\frac{605}{39.37} = 15.367$.

11. How many miles in 12 kilometers? *Ans.* 7.4564.

12. How many kilometers in 15 miles?

Ans. 24.1402.

13. How many quarts (liquid measure) in 41 liters?

Ans. $41 \times 1.0567 = 43.3247$.

14. How many gallons in 4 hectoliters?

Ans. 105.668.

15. How many liters in $31\frac{1}{2}$ gallons?

Ans. $\frac{31.5 \times 4}{1.0567} = 119.2391$.

16. How many ounces in 711 grammes?

Ans. $71.1 \times 0.3527 = 25.077$ *oz. av.*

17. How many pounds in 74 kilogrammes?

Ans. 163.1404.

18. Reduce 510 kilogrammes to pounds.
Ans. 1,124.346 *lbs.*

19. How many cords in 15 steres of wood?
Ans. 4.1385.

20. Reduce $17\frac{1}{2}$ steres of wood to cords.
Ans. 4.8283 *cords.*

21. Reduce 17*s.* $9\frac{3}{4}$*d.* to a decimal of a pound.

22. What part of £1 2*s.* 6*d.* is 1*s.* 11*d.*?

23. Reduce 1 *lb.* 9 *oz.* 15 *grs.* to pounds.

24. Reduce 3 *da.* 14 *hrs.* 25 *min.* to a decimal of 1 week.

25. What part of a barrel is 8 *gals.* 2 *qts.*?

26. What part of a mile is 74 *rds.* 5 *yds.*?

27. What part of a cord is $3\frac{3}{4}$ cord feet?

28. What part of a pound sterling is 5*s.* 9*d.*?

29. Reduce £.187 to lower denominations.

30. Reduce .574 miles to lower denominations.

EXAMINATION QUESTIONS.

(142.) What is reduction? How many kinds of reduction are there? **(143.)** Give the rule for reduction descending. **(144.)** Give the rule for reduction ascending.

III. COMPOUND ADDITION.

Definition.

145. **Compound Addition** is the operation of finding the sum of two or more compound numbers.

The numbers to be added must be similar, that is, they must have a common primary unit.

The principles of compound addition are the same as those of simple addition, (Art. **19**).

Operation of Compound Addition.

146. Let it be required to find the sum of £7 4*s.* 3*d.*, £11 9*s.* 8*d.*, and £14 12*s.* 9*d.*:

OPERATION.

£	*s.*	*d.*
7	4	3
11	9	8
14	12	9
£33	6*s.*	8*d.*

Explanation.—We write the numbers so that units of the same denomination shall stand in the same column. The sum of the numbers in the first column is 20*d.*, or 1*s.* 8*d.*, (Art. **144**); setting down 8*d.*, we *carry* forward 1*s.* and add it to the second column. The sum of the numbers in the second column, thus increased, is 26*s.*, or £1 6*s.*; setting down 6*s.*, we carry forward £1 to the next column, which then amounts to £33. The required sum is therefore £33 6*s.* 8*d.*

In like manner we may treat all similar cases; hence, the following

RULE.

I. Write the numbers so that units of the same denomination shall stand in the same column.

II. Add the units of the lowest denomination and divide their sum by the number of the scale that connects this denomination with the next higher one; set down the remainder and carry the quotient to the next column.

III. Add the units of the second column thus increased, and proceed as before, continuing the operation till all the columns have been added.

EXAMPLES.

Add the following groups of compound numbers:

(1.)

£	*s.*	*d.*
17	13	11
13	10	2
10	17	3
£42	1*s.*	4*d.*

(2.)

cwt.	*qrs.*	*lbs.*
2	3	27
1	1	17
4	2	26
9 *cwt.*	0 *qrs.*	20 *lbs.*

(3.)

℥	℈	*grs.*
3	1	17
2	3	19
6	1	10
13 ℥	1 ℈	6 *grs.*

Proof.—The method of proving compound addition is the same as in addition of simple numbers.

Let the following examples be proved:

(4.)

bu.	*pks.*	*qts.*
17	2	5
34	2	7
13	3	6
16	3	4

(5.)

pks.	*qts.*	*pts.*
3	7	1
2	6	1
0	4	0
3	5	1

(6.)

gals.	*qts.*	*pts.*
2	3	1
5	1	1
7	2	0
11	3	1

(7.)

yds.	*ft.*	*in.*
4	2	11
3	1	8
1	1	9
6	2	1

(8.)

A.	*sq. ch.*	*sq. li.*
5	4	300
2	7	185
9	4	1230
1	8	211

(9.)

da.	*hrs.*	*min.*
4	14	30
3	12	15
5	4	20
6	16	18

(10.)

£	*s.*	*d.*
18	4	9
7	11	6
9	18	9

(11.)

lbs.	*oz.*	*dwts.*
96	10	19
4	6	16
5	10	15

(12.)

mi.	*rds.*	*yds.*
2	120	4
8	72	3
2	112	4

13. Find the sum of 17*s.* 6*d.*; £3 5*s.* 8*d.*; £25 11*s.* $10\frac{1}{2}$*d.*; £12 0*s.* 8*d.*; and £50 4*s.* $4\frac{1}{2}$*d.*

14. Add 37 *bu.* 1 *pk.* 3 *qts.*; 41 *bu.* 2 *pks.* 5 *qts.*; 35 *bu.* 1 *pk.* 3 *qts.*; and 43 *bu.* 3 *pks.* 1 *qt.*

15. It took a carpenter 3 days to build a fence: the first day he built 4 *rds.* 4 *yds.* 1 *ft.* 4 *in.*; the second day he built 3 *rds.* 2 *yds.* 2 *ft.* 9 *in.*; the third day he built 4 *rds.* 3 *yds.* 1 *ft.* 7 *in.* What was the length of the fence?

Reducing to *half yards* and *inches*, (Art. **125**), we have:

rds.	*yds.*	*ft.*	*in.*		*rds.*	*hf. yds.*	*in.*
4	4	1	4	=	4	8	16
3	2	2	9	=	3	5	15
4	3	1	7	=	4	7	1

Ans. 12 *rds.* 10 *hf. yds.* 14 *in.*
= 12 *rds.* 5 *yds.* 1 *ft.* 2 *in.*

If some of the numbers are fractional, they may all be reduced to decimals of the same unit and then added by the rule in Art. **92**.

16. Find the sum of 3*s.* 6*d.*; £$\frac{3}{4}$; and £.875.

17. Add 3 *da.* 16 *hrs.*; $\frac{4}{5}$ of 1 *da.*; and .632 *da.*

18. Add £13 14*s.* 8*d.*; £1 7*s.* $2\frac{1}{2}$*d.*; £3 13*s.* $9\frac{1}{4}$*d.*; £12 12*s.* $3\frac{3}{4}$*d.*; and £17 14*s.* 3*d.*

19. Add 4 *oz.* 15 *dwts.* 12 *grs.*; 3 *oz.* 10 *dwts.* 17 *grs.*; 11 *oz.* 14 *dwts.* 16 *grs.*; and 10 *oz.* 18 *dwts.* 20 *grs.*

20. Add 10 ℥ 7 ʒ 2 ℈ 14 *grs.*; 2 ℥ 4 ʒ 1 ℈ 18 *grs.*; 1 ℥ 3 ʒ 1 ℈ 15 *grs.*; and 3 ʒ 2 ℈ 11 *grs.*

21. Add 7 *cwt.* 3 *qrs.* 14 *lbs.*; 4 *cwt.* 2 *qrs.* 20 *lbs.*; 1 *cwt.* 1 *qr.* 10 *lbs.*; 3 *cwt.* 1 *qr.* 17 *lbs.*; and 13 *cwt.* 0 *qrs.* 18 *lbs.*

22. Add 7 *wks.* 3 *da.* 11 *hrs.*; 5 *wks.* 4 *da.* 19 *hrs.*; 11 *wks.* 2 *da.* 13 *hrs.*; 1 *wk.* 6 *da.* 17 *hrs.*; and 12 *wks.* 4 *da.* 3 *hrs.*

23. Add 2 *mi.* 180 *rds.* 5 *yds.* 2 *ft.*; 3 *mi.* 72 *rds.* 3 *yds.* 1 *ft.*; 8 *mi.* 300 *rds.* 5 *yds.* 1 *ft.*; and 11 *mi.* 18 *rds.* 3 *yds.* 2 *ft.*

24. Add 8 *yds.* 3 *qrs.* 1 *na.*; 7 *yds.* 2 *qrs.* 3 *na.*; 14 *yds.* 2 *qrs.* 3½ *na.*; 11 *yds.* 1 *qr.* 1½ *na.*; and 16 *yds.* 2 *qrs.* 1 *na.*

25. Add 17 *bu.* 3 *pks.* 6 *qts.*; 112 *bu.* 2 *pks.* 7 *qts.*; 91 *bu.*

PRACTICAL PROBLEMS.

1. A merchant sent off the following quantities of butter: 47 *cwt.* 2 *qrs.* 7 *lbs.*; 38 *cwt.* 3 *qrs.* 8 *lbs.*; and 16 *cwt.* 2 *qrs.* 20 *lbs.*; how much did he send off in all?

Ans. 103 *cwt.* 0 *qrs.* 10 *lbs.*

2. A silversmith has 3 parcels of silver: the first contains 7 *lbs.* 8 *oz.* 16 *dwts.*; the second contains 9 *lbs.* 7 *oz.* 3 *dwts.*; and the third contains 4 *lbs.* 1 *dwt.*; how much has he in all? *Ans.* 21 *lbs.* 4 *oz.*

3. A merchant sells cloth as follows: to A., 76 *yds.*, 3 *qrs.* 2 *na.*; to B., 90 *yds.* 3 *qrs.* 3 *na.*; and to C., 190 *yds.* 1 *na.*; how much does he sell?

Ans. 357 *yds.* 3 *qrs.* 2 *na.*

4. A man has three farms: the first contains 120 *A.* 74 *sq. rds.*; the second contains 75 *A.* 46 *sq. rds.*; and the third contains 97 *A.* 110 *sq. rds.*; how much do they all contain? *Ans.* 293 *A.* 100 *sq. rds.*

5. A boy aged 14 *yrs.* 6 *mos.* goes out to service; he lives at one place 1 *yr.* 9 *mos.*, at another place he lives 2 *yrs.* 5 *mos.*, and at a third place he lives 3 *yrs.* 9 *mos.*; how old is he when he leaves the last place?

Ans. 22 *yrs.* 5 *mos.*

EXAMINATION QUESTIONS.

145. What is compound addition? What numbers can be added? **(146.)** Give the rule for adding compound numbers.

IV. COMPOUND SUBTRACTION.

Definition.

147. Compound Subtraction is the operation of finding the difference of two compound numbers.

The minuend and subtrahend must be similar.

The principles of compound subtraction are the same as those of simple subtraction, (Art. **25**).

Operation of Compound Subtraction.

148. Let it be required to find the difference between £9 4*s.* 3*d.* and £2 18*s.* 6*d.*:

OPERATION.

£	*s.*	*d.*
9	4	3
2	18	6
£6	5*s.*	9*d.*

Explanation.—We write the subtrahend under the minuend so that units of the same denomination shall stand in the same column. Beginning at the lowest denomination, we see that 6*d.* cannot be taken from 3*d.*; we therefore add 12*d.* to 3*d.*, which gives 15*d.*, and then subtract 6*d.* from the sum; the remainder, 9*d.*, we set down, and to compensate for the 12*d.* added to the minuend, we add its equal, 1*s.*, to the next column of the minuend. The sum, 19*s.*, being greater than 4*s.*, we add 20*s.* to the latter and subtract 19*s.* from the sum; the remainder, 5*s.*, we set down and as before carry forward 20*s.*, or its equal £1, and add it to the minuend, giving £3; this taken from £9 leaves £6: hence, the required remainder is £6 5*s.* 9*d.*

In like manner we may treat all similar cases; hence, the following

RULE.

I. Write the subtrahend under the minuend so that units of the same denomination shall stand in the same column.

II. Subtract each number in the lower line from the one above it and write the remainder in the line below.

III. If any number in the lower line is greater than the one above it, increase the latter by as many units as make one of the next higher denomination, perform the subtraction and then add 1 unit to the next number in the lower line.

EXAMPLES.

Perform the following indicated subtractions:

	(1.)			(2.)			(3.)		
	£	*s.*	*d.*	*lbs.*	*oz.*	*dwts.*	*bu.*	*pks.*	*qts.*
From	14	14	3	6	11	14	65	1	7
Take	9	17	1	2	3	16	14	3	4
Rem.	£4	17*s.*	2*d.*	4 *lbs.*	7 *oz.*	18 *dwts.*	50 *bu.*	2 *pks.*	3 *qts.*

Proof.—The method of proving compound subtraction is the same as for simple subtraction.

Perform the following indicated subtractions, and let the results be proved

(4.)

cwt.	*qrs.*	*lbs.*
7	3	13
5	1	15

(5.)

hhds.	*gals.*	*qts.*
112	23	1
75	37	1

(6.)

yds.	*ft.*	*in.*
4	2	11
2	2	9

(7.)

acres.	*sq.rds.*
29	50
24	65

(8.)

23°	45′	54″
7°	49′	57″

(9.)

℔	℥	ʒ	℈	*grs.*
35	7	3	1	14
17	10	6	1	18

10. From 4 *rds.* 2 *yds.* 1 *ft.* 9 *in.* subtract 2 *rds.* 3 *yds.* 1 *ft.* 11 *in.*

OPERATION.

rds.	*yds.*	*ft.*	*in.*		*rds.*	*hf.yds.*	*in.*
4	2	1	9	=	4	5	3
2	3	1	11	=	2	7	5

1 *rd.* 8*hf.rds.* 16 *in.*

= 1 *rd.* 4 *yds.* 1 *ft.* 4 *in.* *Ans.*

11. From 12 *rds.* 2 *yds.* 2 *ft.* 1 *in.* subtract 3 *rds.* 3 *yds.* 2 *ft.* 10 *in.* *Ans.* 8 *rds.* 4 *yds.* 9 *in.*

12. From 8 *rds.* 1 *yd.* 2 *ft.* subtract 3 *rds.* 3 *yds.* 2 *ft.* 6 *in.* *Ans.* 4 *rds.* 3 *yds.* 1 *ft.*

To write a date as a compound number we first write the number of the current year, then the number of the current month, counted from the beginning of the year, (Art. **115**), and then the number of the day. Thus.

the date July 7th, 1839, is written 1839 *yrs.* 7 *mos.* 7 *da.*

In computing the difference of two dates, we consider the month equal to 30 days.

13. What is the difference of time between October 16th, 1869, and Aug. 2d, 1873?

OPERATION.

August 2d, 1873 . . .	1873 *yrs.*	8 *mos.*	2 *da.*
October 16th, 1869 . .	1869 *yrs.*	10 *mos.*	16 *da.*
	3 *yrs.*	9 *mos.*	16 *da.* *Ans.*

14. How long is it from September 25th, 1871, to July 4th, 1876? *Ans.* 4 *yrs.* 9 *mos.* 9 *da.*

15. What is the interval between July 7, 1815, and Nov. 1, 1873? *Ans.* 58 *yrs.* 3 *mos.* 24 *da.*

16. How long is it from May 13th, 1816, to June 25th, 1859? *Ans.* 43 *yrs.* 1 *mo.* 12 *da.*

17. The latitude of New York is 40° 42′ 45″ and the latitude of Quebec is 46° 50′ 10″; how many degrees is Quebec to the north of New York?

18. What is the difference between £1.5 and 7*s.* 6*d.*?

19. From $\frac{4}{15}$ of 1 *hhd.* subtract $\frac{4}{7}$ of 1 *qt.*

20. From 3.107 *kilog.* subtract 331.2 *grammes.*

21. From 16 *da.* 21 *hrs.* 42 *min.* 13 *sec.* subtract 12 *da.* 22 *hrs.* 58 *min.* 39 *sec.*

22. From 7 *T.* 14 *cwt.* 3 *qrs.* 19 *lbs.* subtract 3 *T.* 18 *cwt.* 1 *qr.* 7 *lbs.*

23. From 11 *lbs.* 1 *oz.* 3 *dwts.* 18 *grs.* subtract 9 *lbs.* 0 *oz.* 16 *dwts.* 5 *grs.*

24. From 4,306 *gals.* 1 *qt.* subtract 3,621 *gals.* 2 *qts.* 1 *pt.*

25. From 110 *bu.* 1 *pk.* 1 *qt.* subtract 94 *bu.* 3 *pks.* 7 *qts.*

PRACTICAL PROBLEMS.

1. A merchant bought a piece of cloth for £22 10*s.* and sold one half of it for £14 18*s.*; for what must he sell the rest to make £7 14*s.* 3*d.*? *Ans.* £15 6*s.* 3*d.*

2. From a quantity of cloth containing 496 *yds.* 3 *qrs.* 3 *na.* there was sold 247 *yds.* 2 *qrs.* 2 *na.*; how much remained? *Ans.* 249 *yds.* 1 *qr.* 1 *na.*

3. Of a farm containing 273 *A.* 45 *sq. rds.* only 111 *A.* 118 *sq. rds.* was capable of tillage; how much was incapable of tillage? *Ans.* 161 *A.* 87 *sq. rds.*

4. A merchant has 183 *cwt.* 24 *lbs.* of butter, of which he ships 78 *cwt.* 3 *qrs.* 14 *lbs.*; how much remains? *Ans.* 104 *cwt.* 1 *qr.* 10 *lbs.*

5. How long is it from Jan. 20th, 1873, to Nov. 14th, 1875? *Ans.* 2 *yrs.* 9 *mos.* 24 *da.*

EXAMINATION QUESTIONS.

(147.) What is compound subtraction? **(148.)** Give the rule for compound subtraction.

V. COMPOUND MULTIPLICATION.

Definition.

149. Compound Multiplication is the operation of finding the product of two numbers, one of which is compound.

The compound number is taken as the multiplicand; the multiplier is always abstract.

Compound multiplication depends on the same principles as the multiplication of simple numbers, (Art. **31**).

Operation of Compound Multiplication.

150. Let it be required to multiply £4 2*s*. 5*d*. by 16:

Explanation.—Having written the multiplier under the multiplicand, we multiply 5*d*. by 16, which gives 80*d*., or 6*s*. 8*d*.; setting down 8*d*., we carry 6*s*. to the next column. We then multiply 2*s*. by 16 and add 6*s*. to the product, which gives 38*s*., or £1 18*s*.; setting down 18*s*., we carry £1 to the next column. Finally, we multiply £4 by 16 and add £1 to the product, which gives £65. Hence, the required product is £65 18*s*. 8*d*.

OPERATION.

£	*s*.	*d*.
4	2	5
		16
£65	18*s*.	8*d*.

In like manner we may treat all similar cases; hence, the following

RULE.

I. Multiply the units of the lowest denomination of the multiplicand by the multiplier, and divide the product by the number of the scale that connects this denomination with the next higher one; set down the remainder and carry the quotient to the next column.

II. Multiply the units of the next higher denomination by the multiplier, add the units brought forward, and proceed as before, continuing the operation till all the parts of the given number have been multiplied.

EXAMPLES.

Perform the following indicated multiplications:

(1.)

£	*s*.	*d*.
17	15	9
		6
£106	14*s*.	6*d*.

(2.)

cwt.	*qrs*.	*lbs*.	*oz*.
8	3	1	9
			7
61 *cwt*.	1 *qr*.	10 *lbs*.	15 *oz*.

(3.)

wks.	*da.*	*hrs.*	*min.*
1	4	3	25
			11
17	3	13	35

(4.)

bu.	*pks.*	*qts.*
2	1	3
		11
25	3	1

(5.)

cwt.	*qrs.*	*lbs.*	*oz.*
1	3	19	13
			7
13	2	13	11

(6.)

℔	℥	ʒ	℈
1	10	5	2
			5
9	5	4	1

(7.)

mi.	*rds.*	*yds.*	*ft.*	*in.*		*mi.*	*rds.*	*hf. yds.*	*in.*
9	110	4	2	6	=	9	110	9	12
				9					9

84 *mi.* 37 *rds.* 10 *hf. yds.* 0 *in.*
= 84 *mi.* 37 *rds.* 5 *yds.* *Ans.*

8. Multiply 5 *cwt.* 2 *qrs.* 16 *lbs.* by 7.
9. Multiply $8.75 by 24.5.
10. Multiply 65.35 *fr.* by 46.
11. Multiply 5.84 *kilog.* by 12.
12. Multiply 123.25 *meters* by 15.
13. Multiply 15 *yds.* 1 *ft.* by 21.
14. Multiply 17 *gals.* 2 *qts.* by 18.
15. Multiply 6 *wks.* 3 *da.* by 13.
16. Multiply 15 *lbs.* 3 *oz.* 5 *dwts.* by 16.
17. Multiply 13 *yds.* 2 *ft.* 4 *in.* by 9.
18. Multiply £$\frac{2}{3}$ by 16.
19. Multiply 10 *A.* 40 *sq. rds.* by 11.

20. Multiply 3 *hrs.* 15 *min.* 10 *sec.* by 24.
21. Multiply 5 *cwt.* 2 *qrs.* 10 *lbs.* by 24.
22. Multiply £3 14*s.* 10*d.* by 33.
23. Multiply 18 *T.* 2 *qrs.* 16 *lbs.* 9 *oz.* by 48.
24. Multiply £6 12*s.* 4½*d.* by 345.
25. Multiply £7 8*s.* 5*d.* by 648.

PRACTICAL PROBLEMS.

1. If a gentleman spends £1 7*s.* 6*d.* per day, how much does he spend in a year of 365 days?

Ans. £501 17*s.* 6*d.*

2. What is the length of 36 pieces of telegraph wire, each being 26 *mi.* 1,125 *yds.* in length?

Ans. 959 *mi.* 20 *yds.*

3. Find the weight of 37 parcels of silver, averaging 12 *lbs.* 2 *oz.* 15 *dwts.* 6 *grs.* each.

Ans. 452 *lbs.* 6 *oz.* 4 *dwts.* 6 *grs.*

4. How much will it require to give to 324 persons £17 9*s.* 10½*d.* each? *Ans.* £5,667 19*s.* 6*d.*

5. What is the weight of 217 loads, each weighing 3 *T.* 13 *cwt.* 3 *qrs.* 13 *lbs.* gross weight, that is, 28 *lbs.* to the quarter? *Ans.* 801 *T.* 8 *cwt.* 3 *qrs.* 21 *lbs.*

6. How many yards of cloth in 27 bales, each bale containing 15 pieces, and each piece 15 *yds.*?

Ans. 6,075.

EXAMINATION QUESTIONS.

(149.) What is compound multiplication? What must be the nature of the multiplier? **(150.)** Give the rule for compound multiplication.

VI. COMPOUND DIVISION.

Definition.

151. Compound Division is the operation of dividing one number by another, the dividend being compound.

If the divisor is abstract, the quotient is similar to the dividend; if the divisor is similar to the dividend, the quotient is abstract.

The principles of compound division are the same as those of simple division, (Art. **38**).

Operation of Compound Division.

152. Let it be required to divide £65 18*s.* 8*d.* by 16:

Explanation. — Dividing £65 by 16, we find a quotient £4, and a remainder £1. Reducing £1 to shillings and adding 18*s.*, we have 38*s.*, which we take for a new dividend; dividing 38*s.* by 16, we find a quotient 2*s.* and a remainder 6*s.* Reducing 6*s.* to pence and adding 8*d.*, we have 80*d.*, which we take for a new dividend; dividing 80*d.* by 16, we find the quotient 5*d.* and a remainder 0. The required quotient is therefore £4 2*s.* 5*d.*

OPERATION.

```
        £     s.    d.
   16 ) 65    18    8 ( £4 2s. 5d.
        64
        ──
         1 £ . . . 1st rem.
         20
         ──
         38s.
         32
         ──
          6s. . . . 2d rem.
         12
         ──
         80d.
         80
         ──
          0 . . . 3d rem.
```

In like manner we may treat all similar cases; hence, the following

RULE.

I. Divide the units of the highest denomination in the dividend by the divisor and write the quotient as a part of the required quotient.

II. Reduce the remainder to the next lower denomination, to the result add the units of that denomination, for a new dividend, and proceed as before.

III. Continue this operation till the division is completed.

EXAMPLES.

Perform the following indicated divisions:

(1.)

7)37 *bu.* 3 *pks.* 7 *qts.*

Quotient, 5 *bu.* 1 *pk.* $5\frac{4}{7}$ *qts.*

(2.)

9)1 *T.* 19 *cwt.* 2 *qrs.* 12 *lbs.*

4 *cwt.* 1 *qr.* $15\frac{2}{9}$ *lbs.*

Proof.—The method of proof is the same as in simple numbers.

Let the following results be proved:

(3.)

	cwt.	*qrs.*	*lbs.*	*oz.*
7)	17	0	2	6
Quotient. . . .	2	1	18	$3\frac{1}{7}$
				7
Proof.	17	0	2	6

(4.)

	hhds.	*gals.*	*qts.*
12)	9	28	2
		49	$2\frac{1}{2}$
			12
	9	28	2

(5.)

	rds.	*yds.*	*ft.*	*in.*
7)	90	1	1	10

(6.)

	T.	*cwt.*	*qrs.*	*lbs.*	*oz.*
11)	228	18	3	13	12

If the divisor is composite, divide by each factor successively.

(7.)

$$\begin{array}{rrrrl} & \pounds & s. & d. & \\ 24\left\{\begin{array}{r}4)\\6)\end{array}\right. & \begin{array}{r}37\\ \hline 9\end{array} & \begin{array}{r}14\\ \hline 8\end{array} & \begin{array}{r}0\\ \hline 6\end{array} & \\ \cline{2-4} & 1 & 11 & 5 & \textit{Ans.} \end{array}$$

(8.)

$$\begin{array}{rrrrrl} & T. & cwt. & qrs. & lbs. & \\ 90\left\{\begin{array}{r}9)\\10)\end{array}\right. & \begin{array}{r}56\\ \hline 6\end{array} & \begin{array}{r}14\\ \hline 6\end{array} & \begin{array}{r}3\\ \hline 0\end{array} & \begin{array}{r}15\\ \hline 10\end{array} & \\ \cline{2-5} & & 12 & 2 & 11 & \textit{Ans.} \end{array}$$

To divide a compound number by a similar compound number:

RULE.

Reduce both numbers to the same denomination and divide as in simple numbers.

9. $\dfrac{9\,hhds.\ 28\,gals.\ 2\,qts.}{49\,gals.\ 2\,qts.\ 1\,pt.} = \dfrac{4764\,pts.}{397\,pts.} = 12.$ *Ans.*

10. Divide 16 *lbs.* 4 *oz.* (avoirdupois) by 1 *lb.* 10 *oz.*
Ans. 10.

11. Divide 1 *T.* 9 *cwt.* 40 *lbs.* by 4 *cwt.* 20 *lbs.*
Ans. 7.

12. Divide 15 *lbs.* 3 *oz.* 12 *dwts.* by 12.
13. Divide £21 11*s.* 3*d.* by 15.
14. Divide 39 *kilog.* by 7.5.
15. Divide 11 *lbs.* 3 *oz.* by 2 *oz.* 14 *dwts.*
16. Divide 3 miles by 22 *ft.* 6 *in.*
17. Divide 11 *lbs.* 3 *oz.* by 2 *oz.* 10 *dwts.*
18. Divide 1,361 *mi.* 188 *rds.* by 28.
19. Divide 1116 *fr.* 12 *cent.* by 786.
20. Divide 2.03 *hectol.* by 5.8.
21. Divide 655 *lbs.* 8 *oz.* 9 *dwts.* by 17.
22. Divide £30 7*s.* 1¼*d.* by 7.
23. Divide 9 *hhds.* 28 *gals.* 2 *qts.* by 12.

24. Divide 17 *lea.* 1 *mi.* 4 *fur.* 21 *rds.* by 21.
25. Divide 826 *mi.* 217 *rds.* by 17.
26. Divide 64 *sq. yds.* 1 *sq. ft.* 72 *sq. in.* by 132.
27. Divide 178 *lbs.* 9 *oz.* 19 *dwts.* 23 *grs.* by 77.
28. Divide £1,668 15*s.* by £1 2*s.* 3*d.*
29. Divide 3 miles by 2 *ft.* 3 *in.*
30. Divide 186 *fr.* 2 *cent.* by 131.
31. Divide 39 *kilog.* 6 *decag.* by 93.

PRACTICAL PROBLEMS.

1. How many things, costing £2 10*s.* 3*d.* each, can be bought for £40 4*s.*? *Ans.* 16.

2. Divide 794 *wks.* 7 *hrs.* 45 *min.* into 271 equal periods; how long is each?

Ans. 2 *wks.* 6 *da.* 12 *hrs.* 15 *min.*

3. If £145 7*s.* 2¼*d.* is divided equally among 45 men, how much will each receive? *Ans.* £3 4*s.* 7¼*d.*

4. A. bought four fields: the first contained 3 *A.* 25 *sq. rds.*; the second 9 *A.* 57 *sq. rds.*; the third 5 *A* 102 *sq. rds.*; and the fourth 11 *A.* 72 *sq. rds.*; how many acres did he buy?

5. A jeweller bought 11 *lbs.* 6 *oz.* of gold on Monday; 10 *lbs.* 5 *oz.* 20 *grs.* on Tuesday; 6 *oz.* 3 *dwts.* 16 *grs.* on Wednesday; 5 *lbs.* 17 *dwts.* 1 *gr.* on Thursday; 16 *lbs.* 4 *oz.* 15 *dwts.* 15 *grs.* on Friday; and 5 *lbs.* on Saturday; how much did he buy in the week?

6. A farmer had a farm containing 200 acres. He gave to his oldest son 109 *A.* 140 *sq. rds.*; to his second son he gave 48 *A.* 40 *sq. rds.*; and the remainder to his third son; what was the third son's part?

7. A ship must sail 3,000 miles to cross the ocean; if she sails 142 *mi.* 190 *rds.* per day for 12 days, how far has she yet to sail?

8. Four thieves carry off 7 silver cups, each weighing 10 *oz.*; 1 dozen and 9 silver forks, each weighing 2 *oz.* 8 *dwts.*; and 13 silver spoons, each weighing 3 *oz.*; if they divide the silver equally, how much will each have?

9. How many spoons, each weighing 2 *oz.* 10 *dwts.*, can be made from a bar of silver weighing 11 *lbs.* 3 *oz.*?

10. The circumference of the fore wheel of a carriage is 13 *ft.* 9 *in.* and that of the hind wheel 16 *ft.* 6 *in.*; how many more times will the fore wheel turn than the hind one in a journey of 30 miles?

11. If 160 bushels of oysters cost £75 17*s.* 4*d.* what does 1 bushel cost?

12. A truckman carried 117 *C.* 110 *cu. ft.* in 100 loads; how much did he carry per load?

13. If 3 *yds.* of cloth cost £4 16*s.* 6*d.*, how much does 1 yard cost. and how much does 12 *yds.* cost?

14. A person's yearly income is 14,636 *fr.* 80 *c.*; of this he gives in charity 2,500 *fr.*; his weekly bills are 149 *fr.* 15 *c.* each; and the rest he spends in traveling; how much does he spend per week in traveling?

15. The average speed of a railway train is 4 *myriameters* per hour; how long will it take to travel from Paris to Boulogne, a distance of 272 *kilom.*?

16. Bought the following articles: 27½ *meters* of linen at 3 *fr.* 50 *c.* per meter; 6¼ *meters* of velvet at 17 *fr.* per meter; 29 *meters* of brocade at 16 *fr.* 75 *c.* per meter; 28¾ *meters* of merino at 4 *fr.* per meter; 1½ *doz. pairs* of

socks at 3*fr.* 5*c.* per pair; and 7 pairs of gloves at 42*fr.* a dozen pairs; what was the amount of the bill?

17. A person takes 108 steps per minute, each step being 30 inches; how far does he walk in 2*hrs.* 30*min.*?

18. How many yards of cloth will it take to clothe a company of 48 men, if it takes 5⅜ *yds.* for each man?

19. The longitude of New York is 74° 1′ W. and that of Philadelphia is 75° 10′ W.: what is their difference of longitude?

Note.—The *longitude* of a place is the angular distance of the meridian of that place from some fixed meridian, as that of Greenwich. It is measured on the equator, and is equal to the angle through which the earth turns on its axis whilst the sun is passing from the meridian of one place to the meridian of the other place. But the earth turns through an angle of 360° in 24 hours; that is, it turns through 15° *of arc* in 1 *hour*, 15′ *of arc* in 1 *minute of time*, and 15″ *of arc* in 1 *second of time*. Hence, *longitude in arc* may be converted into *longitude in time*, by dividing by 15 and changing the names of the units, as shown below:

Convert longitude 74° 1′ in arc to longitude in time.

Explanation.—Dividing 74 by 15, we find 4*hrs.*, with a remainder equal to 14°, which, reduced to minutes of arc and increased by 1′, gives 841′; this divided by 15 gives 56*min.* and a remainder 1′, or 60″, which divided by 15 gives 4*sec.* In like manner, 75° 10′ is equal to 5*hrs.* 0*min.* 40*sec.*

OPERATION.

15) 74° 1′

4*hrs.* 56*min.* 4*sec.*

20. If 9 sheep cost $42.57, what will 5 sheep cost at the same rate?

EXAMINATION QUESTIONS.

(151.) What is compound division? When will the quotient be similar to the dividend and when abstract? **(152.)** Give the rule for compound division.

CHAPTER V.

RATIO, PROPORTION, AND ANALYSIS.

I. RATIO.

Definition of a Ratio.

153. The **ratio of one number to another** is the quotient obtained by dividing the second by the first. Thus, the ratio of 2 to 4 is equal to $\frac{4}{2}$.

The *first* number is called the *antecedent*, the *second* is called the *consequent*, and both are called *terms* of of the ratio. In the ratio just given, 2 is the *antecedent*, 4 is the *consequent*, and both 4 and 2 are *terms*.

Methods of Expressing a Ratio.

154. A ratio may be written in the form of a fraction, as already explained, or it may be expressed by writing the consequent after the antecedent with a *colon* between them. Thus, the ratio of 2 to 4 may be written $\frac{4}{2}$, or it may be written 2 : 4; in either case it may be read: *the ratio of* 2 *to* 4, or 2 *is to* 4.

The terms of a ratio, with their connecting sign, form a *couplet;* thus, the expression 2 : 4 is a couplet.

155. From the definition of a ratio we have the following principles:

1°. *A ratio is equal to the consequent divided by the antecedent.*

2°. *The consequent is equal to the antecedent multiplied by the ratio.*

3°. *The antecedent is equal to the consequent divided by the ratio.*

Either of the three numbers, the *ratio*, the *antecedent*, or the *consequent*, can be found if the other two are given, or known.

EXAMPLES.

1. What is the ratio of 3 to 5? *Ans.* $\frac{5}{3} = 1\frac{2}{3}$.

2. What is the ratio of 7 to 11? *Ans.* $\frac{11}{7} = 1\frac{4}{7}$.

3. What is the ratio of 3*s.* 6*d.* to 17*s.* 6*d.*? *Ans.* 5.

4. What is the ratio of 4 *ft.* 8 *in.* to 2 *ft.* 4 *in.*? *Ans.* $\frac{1}{2}$.

5. What is the ratio of 14 *lbs.* 6 *oz.* to 43 *lbs.* 2 *oz.*? *Ans.* 3.

6. What is the ratio of 27 *bu.* 1 *pk.* to 6 *bu.* 3 *pks.* 2 *qts.*? *Ans.* $\frac{1}{4}$.

7. The antecedent is 7 and the ratio 4; what is the consequent? *Ans.* 28.

8. The antecedent is $\frac{1}{2}$ and the ratio $\frac{1}{8}$; what is the consequent? *Ans.* $\frac{1}{16}$.

9. The antecedent is 5 *lbs.* 4 *oz.* and the ratio $\frac{1}{2}$; what is the consequent? *Ans.* 2 *lbs.* 10 *oz.*

10. The consequent is 18 *lbs.* 6 *oz.* and the ratio is 6; what is the antecedent? *Ans.* 3 *lbs.* 1 *oz.*

11. The consequent is 105 and the antecedent is $7\frac{1}{2}$; what is the ratio? *Ans.* 14.

12. The consequent is £37 11*s.* 6*d.* and the ratio is 9; what is the antecedent? *Ans.* £4 3*s.* 6*d.*

13. What is the ratio of 3 *da.* 10 *hrs.* to 4 *wks.* 6 *da.* 4 *hrs.*? *Ans.* 10.

14. The consequent is 12 and the antecedent is 4; what is the ratio? *Ans.* 3.

15. The consequent is 7 and the ratio $\frac{3}{4}$; what is the antecedent? *Ans.* $\frac{28}{3} = 9\frac{1}{3}$.

EXAMINATION QUESTIONS.

(153.) What is the ratio of one number to another? Define antecedent and consequent. **(154.)** How may a ratio be expressed? What is a couplet? **(155.)** What are the principles used in treating ratios?

II. Proportion.

Definition of a Proportion.

156. A proportion is an expression of equality between two equal ratios; thus,

$$2 : 3 = 16 : 24$$

is a proportion; it is read, *the ratio of* 2 to 3 *is equal to the ratio of* 16 *to* 24.

The sign of equality is usually replaced by a double colon; thus, the preceding proportion may be written:

$$2 : 3 :: 16 : 24.$$

It is then read, 2 *is to* 3 *as* 16 *is to* 24.

Names of Terms.

157. The *terms* of a proportion are called by different names, according to the places they occupy:

The *first* and *third* terms are called *antecedents*; the *second* and *fourth*, *consequents*; the *first* and *fourth*, *extremes*; the *second* and *third*, *means*; the *fourth* term is called a *fourth proportional* to the *other three.*

The *first* and *second* terms form the *first couplet*; the *third* and *fourth* terms, the *second couplet.*

Solution of a Proportion.

158. The **Solution of a Proportion** is the operation of finding one of its terms, when the other three are known.

The rule by which the solution is made is called the *rule of three.*

Principles of Proportion.

159. The solution of proportions depends on the following principles:

1°. *If both terms of either ratio are multiplied, or divided, by the same number the proportion will not be changed.*

For, if both terms of a ratio are multiplied, or divided, by the same number its value will not be changed, (Prin. 5° and 6°, Art. **65**).

2°. *The product of the extremes is equal to the product of the means.*

For, any proportion, as,

$$3 : 6 :: 8 : 16,$$

may be written:

$$\frac{6}{3} = \frac{16}{8};$$

multiplying both terms of the first ratio by 8, and both terms of the second ratio by 3, (Prin. 2°), we have,

$$\frac{6 \times 8}{3 \times 8} = \frac{16 \times 3}{8 \times 3};$$

because these fractions are equal and have equal denominators, their numerators must be equal, that is,

$$16 \times 3 = 6 \times 8;$$

which was to be shown.

From principle 2°, we deduce the two following:

3°. *Either extreme is equal to the product of the means divided by the other extreme.*

4°. *Either mean is equal to the product of the extremes divided by the other mean.*

160. In applying the preceding principles to the solution of proportions, it is customary to represent the required term by some letter, as x, and the couplet into which this term enters is written after the other.

EXAMPLES.

1. Let it be required to solve the proportion,

$$15 : 45 :: 9 : x$$

In this case the required term is one of the extremes; hence, from principle 3°, we have,

$$x = \frac{45 \times 9}{15} = 27. \quad \textit{Ans.}$$

After indicating the solution, we strike out all factors common to both numerator and denominator.

2. Let it be required to solve the proportion,

$$\$7 : \$13 : \ 56\,\textit{lbs.} : x.$$

$$\textit{Ans.} \ \ x = \frac{13 \times 56\,\textit{lbs.}}{7} = 104\,\textit{lbs.}$$

In solving a proportion, if the first two terms are denominate, we always disregard their common unit.

In the example just given, it is obvious that the ratio of $7 to $13 is the same as the ratio of 7 to 3.

Find the value of x in each of the following examples:

3. $\$15 : \$3 :: x : 4\,\textit{yds.}$ *Ans.* $x = 20\,\textit{yds.}$
4. $2 : 3 :: 18 : x.$ *Ans.* $x = 27.$
5. $8 : 32 :: 24 : x.$ *Ans.* $x = 96.$
6. $32 : 18 :: 16 : x.$ *Ans.* $x = 9.$
7. $8 : 4 :: \frac{1}{3} : x.$ *Ans.* $x = \frac{1}{6}.$
8. $1.2 : 6 :: x : 1.3.$ *Ans.* $x = .26.$
9. $5\,\textit{feet} : 7\,\textit{feet} :: \$3 : x.$ *Ans.* $x = \$4.20.$
10. $9\,\textit{days} : 15\,\textit{days} :: £2\ 2s. : x$ *Ans.* $x = £3\ 10s.$
11. $30\,\textit{tons} : 12\,\textit{tons}\ 10\,\textit{cwt.} :: \$650 : x.$

    Ans. $x = \$270.83\frac{1}{3}.$
12. $28\,\textit{barrels} : 1\frac{1}{2}\,\textit{barrels} :: \$140 : x.$ *Ans.* $x = \$7.50.$
13. $25\,\textit{sq. yds.} : 14\frac{3}{4}\,\textit{sq. yds.} :: £7\ 10s. : x.$

    Ans. $x = £4\ 8s.\ 6d.$

14. $84.50 : $21.12½ : : 13 *cords* : x.
Ans. $x = 3\frac{1}{4}$ *cords.*

15. 75 *bu.* : 4 *bu.* 3 *pk.* : : x : $10.50.
Ans. $x = \$165.78\frac{18}{19}$.

16. £1 2*s.* 6*d.* : £7 17*s.* 6*d.* : : 7 : x. *Ans.* $x = 49$.

17. 12 *lbs.* : 30 *lbs.* : : $2 : x. *Ans.* $x = \$5$.

18. 17 *bu.* : 43 *bu.* : : 25½ : x. *Ans.* $x = 64\frac{1}{2}$.

19. 100 *bu.* : 1 *bu.* : $150 : x. *Ans.* $x = \$1\frac{1}{2}$.

20. 36 *gals.* : 21 *gals.* : : $90 : x. *Ans.* $x = \$52\frac{1}{2}$.

21. 44 : 40 : : $23 : x. *Ans.* $x = \$20\frac{10}{11}$.

Rule of Three.

161. The rule of three is a rule for finding from *three* numbers a *fourth*, to which the *third* shall have the same ratio as the *first* has to the *second*.

This rule depends on the principles of Articles **155** and **159**.

Operation.

162. Let it be required to solve the following problem:

If 40 *yds.* of cloth cost $170, what will 64 *yds.* cost?

Explanation.—We first *state the problem*, that is, we denote the required number by x, and then express the conditions of the problem in the form of a proportion.

SOLUTION.

40 *yds.* : 64 *yds.* : : $170 : x.

$$\begin{array}{r} 170 \\ 40\overline{)10880} \\ \hline \$272 \end{array} \quad \textit{Ans.}$$

Having written x for the *fourth* term, we write the number having the same unit, that is, $170, for the *third* term. We then consider whether the *fourth* term is greater, or less than the third; in this example, it is plain that 64 yards will cost more than 40 yards, that is, the fourth term is greater than the third; we therefore write the smaller of the remaining numbers for the *first* term, and the *greater one* for the second term. Having completed the statement, we solve the resulting proportion as explained in Article **160.**.

Since all similar cases may be treated in like manner, we have the

RULE OF THREE.

I. Denote the required number by x, and write it for the fourth term of a proportion; then write the number that has the same unit for the third term.

II. Consider, from the nature of the question, whether the fourth term will be greater, or less than the third, and write the remaining numbers, in the same relative order, for the first and second terms.

III. Solve the resulting proportion, and the value of x will be the answer.

EXAMPLES.

1. If 25 yards of silk cost $81.25, what will 37 yards cost?

It is plain that 25 yards must bear the same relation to 37 yards that the cost of 25 *yds.* does to the cost of 37 *yds.*; hence, from the nature of the case, and in accordance with the rule, we have the proportion,

$$25\ yds. : 37\ yds. :: \$81.25 : x;$$

Solving, we have, $x = \$120.25$. *Ans.*

2. If a man can walk 84 miles in 3 days, how far can he walk in 11 days?

Statement. $3\ days : 11\ days :: 84\ miles : x;$

$$\therefore\ x = 308\ miles. \quad Ans.$$

The sign ∴ stands for *hence*.

3. If it costs £2 9*s.* 6*d.* to travel 198 miles, how far can I travel for £8 0*s.* 10½*d.*?

Statement. £2 9*s.* 6*d.* : £8 0*s.* 10½*d.* : : 198 *mi.* : x;
$\therefore x = 643\frac{1}{2}$ *miles.* *Ans.*

4. If 12 *lbs.* 3 *oz.* of gold cost $2878.75, what will 3 *oz.* cost? *Ans.* $58.75.

5. If 31 *cwt.* 1 *qr.* 14 *lbs.* of sugar cost $318.45, what will 1,240 *lbs.* cost? *Ans.* $125.7974.

If all the numbers have the same unit, the nature of the question will show which is to be taken as the third term.

6. If a piece of property worth $3,250 is taxed $35.75, what should be the tax on a house worth $17,350?

Here the answer is to be the tax on $17,350; hence, the third term must be the tax on $3,250.

Statement. $3,250 : $17,350 : : $35.75 : x.
$\therefore x =$ $190.85. *Ans.*

7. Find a fourth proportional to 3, 4, and 21. *Ans.* 28.

8. If 3 pairs of stockings cost $1.41, what will 7 pairs cost? *Ans.* $3.29.

9. If 4½ tons of hay will keep 2 cows for the winter, how many cows can be kept on 24¾ tons? *Ans.* 11.

10. If 18¾ bags of coffee contain 758 *lbs.* 8 *oz.*, how many bags are there in 12,136 *lbs.*? *Ans.* 300.

11. How long will it take to travel 1,290 miles, at the rate of 306¾ miles in 20½ days? *Ans.* $86\frac{86}{409}$ *days.*

12. If 24 *yds.* 3 *qrs.* of carpet, 1 yard wide, will cover a room, how many *yds.* of carpet $1\frac{3}{4}$ *yds.* wide will it take to cover the same room?

In this case it is plain that it will take fewer yards of the latter width than of the former; hence, the fourth term is less than the third.

Statement. $1\frac{3}{4}$ *yds.* : 1 *yd.* : : 24 *yds.* 3 *qrs.* : x;

$\therefore x = 14\frac{1}{7}$ *yds.* *Ans.*

13. If 12 men can build a wall in 20 days, how many men would it take to build it in 5 days?

It would require more men to build it in 5 than in 20 days; hence, the fourth term is greater than the third.

Statement. 5 *da.* : 20 *da.* : : 12 *men* : x;

$\therefore x = 48$ *men.* *Ans.*

14. If a piece of cloth 20 yards long and $\frac{3}{4}$ of a yard wide is required to make a riding habit, what must be the width of a piece 12 yards long to make the same habit?

Statement. 12 : 20 : : $\frac{3}{4}$ *yd.* : x;

$\therefore x = 1\frac{1}{4}$ *yds.* *Ans.*

15. In what time can 25 men do a piece of work that 12 men can do in 3 days?

Statement. 25 *men* : 12 *men* : : 3 *days* : x;

$\therefore x = 1\frac{11}{25}$ *days.* *Ans.*

16. A. exchanged 60 yards of silk, worth $2.40 per yard, for 48 yards of velvet; what did the velvet cost per yard?

Ans. $3.

17. If 30 bushels of oats, at 50 cents a bushel, will pay a debt, how much barley must be given to pay the same debt, barley being worth 75 cents a bushel?

Ans. 20 *bushels.*

18. If 42 tons of coal cost $197.40, what will $1\frac{3}{4}$ tons cost? *Ans.* $8.225.

19. If a man can do a piece of work in 20 days, working 10 hours per day, how long will it take him to do the same if he works 12 hours per day?

Ans. $16\frac{2}{3}$ *days.*

20. If $17\frac{7}{9}$ cords of wood cost $\$88\frac{8}{9}$, what will $254\frac{3}{4}$ *cu.ft.* cost? *Ans.* $9.95.

21. If $7\frac{7}{11}$ barrels of apples cost $\$31\frac{1}{9}$, what will $32\frac{2}{5}$ barrels cost? *Ans.* $132.

22. If 2 *bu.* 1 *pk.* of wheat cost $\$1.93\frac{3}{4}$, how many bushels can be bought for $\$96\frac{7}{8}$? *Ans.* $112\frac{1}{2}$ *bushels.*

23. If 10 bushels of coal cost 25 *fr.* 50 *c.*, what will 13 bushels cost? *Ans.* 33.15 *fr.*

24. If 14 meters of cloth can be bought for 350 *fr.*, how much can be bought for 875 *fr.* *Ans.* 35 *meters.*

25. If it cost $40 to board 3 men 5 weeks, what will it cost to board 12 men 10 weeks?

It is plain that the board of 3 men for 5 weeks is the same as the board of 1 man for 15 weeks, and that the board of 12 men for 10 weeks is the same as the board of 1 man for 120 weeks. By the rule we have the following statement:

$$15 \textit{ weeks' board} : 120 \textit{ weeks' board} :: \$40 : x;$$

$$\therefore x = \$320. \quad \textit{Ans.}$$

26. If 18 men consume 34 barrels of potatoes in 135 days, how long will it take 45 men to consume 102 barrels?

The part of a barrel that each man consumes in the first case is $\frac{34}{18}$, and in the second case it is $\frac{102}{45}$; in the first case the time is 135 days and in the second case it is x days; here x is obviously greater than 135 days, for it will take longer to consume $\frac{102}{45}$ *bbl.* than to consume $\frac{34}{18}$ *bbl.*; hence, the fourth term is greater than the third, and, consequently, we have the

Statement. $\frac{34}{18}$ *bbl.* : $\frac{102}{45}$ *bbl.* :: 135 *da.* : x;

$\therefore x = 162$ *da.* ***Ans.***

27. If 12 boys pay $2,000 for 1 year's tuition, what must 14 boys pay for $18\frac{1}{2}$ months' tuition?

Statement. 12×12 *mo.* : $14 \times 18\frac{1}{2}$ *mo.* :: $2,000 : x;

$\therefore x = \$3{,}597.22\frac{2}{9}$. ***Ans.***

28. If the transportation of 18 *cwt.* 2 *qrs.* for $5\frac{1}{2}$ miles cost $7.20, what will it cost to transport 112 *T.* 15 *cwt.* for $62\frac{1}{2}$ miles?

Statement.

18 *cwt.* 2 *qr.* $\times 5\frac{1}{2}$: 112 *T.* 15 *cwt.* $\times 62\frac{1}{2}$ :: $7.20 : x;

$\therefore x = \$9{,}972.973$. ***Ans.***

29. If 20 men working 11 hours a day for 30 days can earn $3,300, how much can 36 men earn in 40 days working 10 hours per day?

Statement. 20 . 30 . 11 : 36 . 40 . 10 :: $3,300 : x;

$\therefore x = \$7{,}200$. ***Ans.***

30. If 7 men reap 6 acres in 12 hours, how many men must be employed to reap 15 acres in 14 hours?

Statement. $\frac{6}{12} A. : \frac{15}{14} A. :: 7\,men : x.$

$\therefore x = 15$ men. *Ans.*

31. If 14 horses eat 56 bushels of oats in 16 days, how many horses will it take to eat 120 bushels in 24 days?

Statement. $\frac{56}{16}\,bu. : \frac{120}{24}\,bu. :: 14\,horses : x;$

$\therefore x = 20\,horses.$ *Ans.*

32. If 12 horses can plough 11 acres in 5 days, how many horses would be required to plough 33 acres in 18 days?

Statement. $\frac{11}{5} A. : \frac{33}{18} A. :: 12\,horses : x;$

$\therefore x = 10\,horses.$ *Ans.*

33. If a man can walk 250 miles in 9 days of 12 hours each, how many days of 10 hours each would it take him to walk 400 miles? *Ans.* $17\frac{7}{25}$ *days.*

34. If 3,000 copies of a book of 11 sheets require 66 reams of paper, how much paper will 5,000 copies of a book of $12\frac{1}{2}$ sheets require? *Ans.* 125 *reams.*

35. If 24 men can reap 76 acres in 6 days, how long will it take 18 men to reap 114 acres? *Ans.* 12 *days.*

36. If 10 men can blast out 30 cubic yards of rock in 8 days, how many cubic yards can 20 men blast out in 10 days? *Ans.* 75 *cubic yards.*

37. If 7 men can mow 84 acres in 12 days of $8\frac{1}{4}$ hours each, in how many days of $7\frac{4}{5}$ hours each can 20 men mow 208 acres? *Ans.* 11 *days.*

Distributive Proportion.

The *rule of three* enables us to divide a number into parts proportional to two or more given numbers:

38. Let \$140 be divided into three parts, proportional to 3, 5, and 6.

The sum of the numbers to which the parts are proportional is 14. Now it is obvious that 140 bears the same relation to the first part that 14 bears to 3; hence, we have,

14 : 3 :: \$140 : the *first part.*

In like manner, we have,

14 : 5 :: \$140 : the *second part.*
14 : 6 :: \$140 : the *third part.*

Solving these proportions, we have,

$$\left.\begin{array}{l}\text{1st part} = \$30\\ \text{2d part} = \$50\\ \text{3d part} = \$60\end{array}\right\}\textit{Ans.}$$

39. Divide 20 *lbs.* 4 *oz.* into 3 parts proportional to 3. 5, and 10.

Ans. 3 *lbs.* 6 *oz.*; 5 *lbs.* 10 *oz.*; and 11 *lbs.* 4 *oz.*

40. Divide \$540 among a man, his wife and three children, so that the wife shall have twice as much as each child, and the man twice as much as his wife.

$$\textit{Ans.}\left\{\begin{array}{ll}\text{Man's share,} & \$240.\\ \text{Wife's share,} & \$120.\\ \text{Each child's share,} & \$60.\end{array}\right.$$

41. A. and B. start from places [illegible] miles apart and travel towards each other; A. travels [illegible] miles per hour, and B. travels 8 miles per hour. How far does each travel before they meet.

Ans. { A. travels [illegible] miles.
B. travels 80 miles.

The methods just explained are applicable to the distribution of gain or loss in partnership.

42. A., B., and C. enter into partnership; A. puts in $720, B. puts in $340, and C. puts in $960; if they gain $505, how much should each receive?

Ans. A., $180; B., $85; and C., $240.

43. A. and B. buy goods to the amount of $400, of which A. pays $150 and B. $250; if they lose $100, how much of the loss must each bear?

Ans. A. loses $37½, and B. $62½.

44. A., B., and C. engage in a speculation towards which A. contributes $480, B. contributes $720, and C. contributes $1,200; if they gain $650, what does each gain? *Ans.* A. gains $130; B. $195; and C. $325.

45. A bankrupt owes A., $500, B., $750, C., $900, and D., $1,250, but his estate is worth only $1,020; what share ought each to receive?

Ans. A., $150; B., $225; C., $270; and D., $375.

EXAMINATION QUESTIONS.

(156.) What is a proportion? Illustrate. What are the signs of proportion? **(157.)** What names are given to the different terms? Which are the antecedents? The consequents? Extremes? Means? **(158.)** What is the solution of a proportion? What is the rule for solving proportions called? **(159.)** On what principles does it depend? **(160.)** Explain the method of solving a proportion. **(161.)** What is the rule of three?

III. ANALYSIS.

Definition.

163. Analysis is the method of solving problems by the direct application of general principles, without the use of particular rules.

Most of the problems that are usually solved by the rule of three can also be solved, and often more expeditiously, by analysis.

The method of proceeding will be shown best by examples.

EXAMPLES.

1. If 34 men can build a house in 40 days, how long will it take 12 men to build the same house?

OPERATION.

$$\frac{34 \times 40 \textit{ da.}}{12} = 113\tfrac{1}{3} \textit{ da.} \quad \textit{Ans.}$$

Analysis.—It will take 1 man 34 times as long as it will 34 men; hence, it will take 1 man 34 × 40 days. But 12 men can build it in $\frac{1}{12}$ of the time that 1 man can; hence, 12 men can build it in $\frac{34 \times 40 \text{ days}}{12}$, or in $113\frac{1}{3}$ *days.*

2. If 2 *cwt.* 3 *qrs.* 10 *lbs.* of sugar cost $34.20, how much can be bought for $75.60?

OPERATION.

$$\frac{\$34.20}{285} = 12 \textit{ cts.}, \text{ cost of } 1 \textit{ lb.}$$

$$\frac{\$75.60}{12 \textit{ cts.}} = 630, \textit{ number of lbs.}$$

$$\therefore \frac{75.60 \times 285 \textit{ lbs.}}{34.20} = 630 \textit{ lbs.} \quad \textit{Ans.}$$

Analysis.—If we divide $34.20 by 285, the number of lbs., the quotient will be the cost of 1 *lb.* If we divide $75.60 by the quotient, 12 *cts.*, the resulting quotient will be the required number of pounds. The entire operation is indicated in the last line.

3. If 6 men can reap 80 acres in 12 days, how many days will it take 25 men to reap 200 acres?

Analysis.—If it takes 6 men 12 days to reap 80 acres, it will take 1 man 6×12 days to do the same; hence, 1 man can reap $\frac{80}{6 \times 12}$ acres in 1 day, and, consequently, 25 men can reap 25 times as much in 1 day, that is, they can reap $\frac{80 \times 25}{6 \times 12}$ acres; now, if we divide 200 *acres* by the number of acres that 25 men can reap in one day, the quotient, $7\frac{1}{5}$, will be the required number of days.

OPERATION.

$$200 \div \frac{80 \times 25}{6 \times 12}$$

$$= \frac{200 \times 6 \times 12}{80 \times 25} = 7\tfrac{1}{5};$$

∴ $7\frac{1}{5}$ days. *Ans.*

4. A. can do a piece of work in 4 days, and B. can do it in 6 days; how long will it take them to do it, if they work together?

Explanation.—A. can do $\frac{1}{4}$ of the work in 1 day, and B. can do $\frac{1}{6}$ of it in the same time; hence, both together can do $\frac{1}{4}+\frac{1}{6}$, or $\frac{5}{12}$ of it in 1 day; but if they can do $\frac{5}{12}$ of it in 1 day, they can do $\frac{1}{12}$ of it in $\frac{1}{5}$ of 1 day, and, consequently, they can do $\frac{12}{12}$ of it in $\frac{12}{5}$ days, that is, in $2\frac{2}{5}$ days.

OPERATION.

$$\frac{1}{4} + \frac{1}{6} = \frac{5}{12};$$

$$1 \div \frac{5}{12} = 2\tfrac{2}{5};$$

∴ $2\frac{2}{5}$ days. *Ans.*

5. Three men hire a pasture for \$45; the first puts in 3 horses for 5 weeks, the second puts in 4 horses for 3 weeks, and the third puts in 7 horses for 4 weeks; what should each man pay?

Explanation.—Since the pasturage of 3 horses for 5 weeks is equivalent to the pasturage of 1 horse for 15 weeks, the first received the benefit of 15 weeks' pasturage for 1 horse; the second, in like manner, received 12 weeks' pasturage

OPERATION.

1st. $3 \times 5 = 15$; $\frac{15}{55}$ of \$45 = \$$12\frac{3}{11}$;
2d. $4 \times 3 = 12$; $\frac{12}{55}$ of \$45 = \$$9\frac{9}{11}$;
3d. $7 \times 4 = 28$; $\frac{28}{55}$ of \$45 = \$$22\frac{10}{11}$.
Sum, 55

for 1 horse; and the third received 28 weeks' pasturage for 1 horse; they all received 55 weeks' pasturage for 1 horse. Hence, the first should pay $\frac{15}{55}$, the second $\frac{12}{55}$, and the third $\frac{28}{55}$ of the rent.

In like manner other problems may be analyzed and solved. Let all the examples in the last article be solved by analysis, and also the following:

6. If 2 *bu.* 1 *pk.* of wheat cost $2.43, what will 14¼ *bu.* cost? *Ans.* $15.39.

7. If 14 men can board 1 week for $45.50, how long can 3 men board for $97.50? *Ans.* 10 *weeks.*

8. If a steamer sails 728 miles in 2⅓ days, how far will she sail in 12½ days? *Ans.* 3,900 *miles.*

9. If 20 men perform a piece of work in 12 days, how many men will be required to do a piece of work 3 times as great in ⅕ of the time? *Ans.* 300.

10. If 450 *lbs.* of coffee cost $99, how much will 1,450 *lbs.* cost? *Ans.* $319.

11. If 27 tons of iron cost $540, how much will 37½ tons cost? *Ans.* $750.

12. If 17 bushels of wheat are worth $25.50, how much are 29 bushels worth? *Ans.* $43.50.

13. If 3 dozen of wine cost $28.50, how much will 5½ dozen cost? *Ans.* $52.25.

14. If 117 bushels of barley cost $105.30, how much will 413 bushels cost? *Ans.* $371.70.

15. If 36 gallons of molasses cost $32.40, how much can be bought for $105.30? *Ans.* 117 *gals.*

16. If I pay $3.00 for riding 40 miles in a stage-coach, how far can I ride for $10.42½? *Ans.* 139 *miles.*

17. If 32 acres of land can be bought for $1,504, how much can be bought for $5,546? *Ans.* 118 *acres.*

18. Find the cost of $25\frac{1}{2}$ *lbs.* of tea, when 17 *lbs.* can be bought for $15.30. *Ans.* $22.95.

19. If 11 Irish miles are equal to 14 English miles, what is the length, in English miles, of a road that measures 57 Irish miles? *Ans.* $72\frac{6}{11}$ *English miles.*

20. If a staff 4 feet high casts a shadow 6 feet long, what must be the height of a pole that will cast a shadow 58 feet long at the same time? *Ans.* $38\frac{2}{3}$ *ft.*

21. A man paid $36 to several laborers; to each man he paid $4, and to each boy $2; the number of men was equal to the number of boys; how many were there of each? *Ans.* 6.

22. If 14 men consume $20 worth of flour in 15 days, how many days will it last 21 men? *Ans.* 10 *days.*

23. If a barrel of flour will make 180 ten-cent loaves, how many eight-cent loaves will it make? *Ans.* 225.

24. A horse and saddle together were worth $100, and the horse was worth 9 times as much as the saddle; what was the horse alone worth? *Ans.* $90.

25. A farmer puts a flock of sheep in 3 pastures; in the first he puts $\frac{1}{3}$ of his flock, in the second $\frac{1}{2}$ of his flock, and in the third he puts 32 sheep; how many sheep has he? *Ans.* 192.

26. What number is that to which if its sixth part and its eighth part be added the sum will be 186?

Ans. 144.

27. A woman buys eggs at the rate of 3 for 5 cents, and sells them at the rate of 4 for 7 cents, clearing 9 cents by the bargain; how many eggs does she buy?

Ans. 108.

28. A farmer gave 5 loads of straw for 12 tons of coal, worth $\$6\frac{1}{2}$ per ton; what did he get per load for his straw? *Ans.* $\$15\frac{3}{5}$.

29. At what time between 1 and 2 o'clock are the hands of a watch together?

Analysis.—The hands of a watch are together at 12 o'clock, and the hour hand gains 55 minute spaces in an hour; but it must gain 60 minute spaces before they can be together; hence, the time required is $\frac{60}{55}$ *hrs.*, or 1 *hr.* $5\frac{5}{11}$ *min.*, that is, they are together at $5\frac{5}{11}$ minutes past 1. *Ans.*

30. If sugar is worth $7\frac{1}{2}$ cents per lb., how many pounds can be bought for 2 *tons* 10 *cwt.* of iron, at $60 per ton? *Ans.* 2000.

31. How many acres of land, at $150 per acre, must be given for 750 *bbls.* of flour, at $4.60 per barrel? *Ans.* 23.

32. How many kilogrammes of butter, at 50 *ct.*, a *kilog.*, must be given for 7 *meters* of cloth, at $4.50 per meter? *Ans.* 63.

33. How many meters of cloth, at 20 *fr.* per meter, must be given for a watch worth 315 *fr.* 20 *c.*? *Ans.* 15.76.

34. If 3 trees furnish 8.1 *steres* of timber, what will 62 trees of the same kind produce? *Ans.* 167.4 *steres.*

35. If 5 children at a picnic eat 21 *hectogrammes* of cake, how much will it take to feed 93 children? *Ans.* 39.06 *kilogrammes.*

36. Divide 55 *fr.* 35 *c.* among 7 men and 6 women, giving to each man 3 times as much as to each woman.

Ans. Each man has 6 *fr.* 15 *c.*, and
Each woman, 2 *fr.* 05 *c.*

37. Divide 203 *liters* of soup among 18 men and 22 women, giving to each man twice as much as to each woman. *Ans.* Each man's share, 7 *liters*;
Each woman's share, 3.5 *liters.*

Problems relating to the distribution of loss or gain among partners may be solved like example 5.

38. Two men enter into partnership; the first puts in $60, the second $80, and they gain $35; what is the gain of each? *Ans.* 1st, $15; 2d, $20.

39. A., B., and C. enter into speculation; A. puts in $4,000, B. puts in $5,000, and C. puts in $6,000; they lose $2,000; what part of the loss must each bear?

Ans. A. loses 533.33\frac{1}{3}$; B. 666.66\frac{2}{3}$; and C. $800.

40. Three men hire a pasture for 1 year and pay $45 for its use; the first puts in 100 head of cattle, the second puts in 150 head, and the third puts in 50 head; what must each pay?

Ans. The first must pay $15, the second $22.50, and the third $7.50.

41. A.'s capital is $3,150, B.'s is $5,000, and C.'s is $4,560, and they gain $2,542; what is the gain of each?

Ans. A. gains $630, B. gains $1,000, and C. gains $912.

42. S., J., and B. enter into partnership; S. puts in $5,600, J. $4,900, and B. $3,500; if they gain $1,650, how much will each gain?

Ans. S. will gain $660, J. $577.50, and B. $412.50.

43. A. and B. commence trade; A. puts in $350 for 8 months, B. $600 for 7 months, and they make $700; to what part of the gain is each entitled?

Solution.—A.'s capital is equivalent to 8 × $350, or $2,800 for 1 month, and B.'s capital is equivalent to 7 × $600, or $4,200 for 1 month. Dividing the gain in the proportion of $2,800 and $4,200, we find for A.'s share $280 and for B.'s share $420.

44. W. and B. engage in business: W. puts in $18,000 for 17 months, and B. puts in $24,000 for 6 months; while in business they lose $6,500; how much of the loss must each bear?

Ans. W. loses $4,420, and B. $2,080.

45. On the 1st of January, 1873, A. commenced business with a capital of $17,000; on the 1st of April B. entered into the business, advancing $12,000 capital; and on the 1st of July C. was admitted as a partner and advanced $16,000 capital; at the end of the year it was found that the firm had gained $8,160. How much of the gain ought each to receive?

Ans. A. should receive $4,080, B. $2,160,
and C. $1,920.

46. A., B., and C. have business transactions together whereby they gain $18,049.60; A. furnished $22,000 for 12 months, B. $18,600 for 10 months, and C. $30,000 for 7 months; to what part of the gain is each entitled?

Ans. A. is entitled to $7,219.84; B. to
$5,086.70$\frac{6}{11}$, and C. to $5,743.05$\frac{5}{11}$.

EXAMINATION QUESTIONS.

(163.) What is analysis? What is the general method of solving a question by analysis?

CHAPTER VI.

PERCENTAGE, AND ITS APPLICATIONS TO BUSINESS.

I. Percentage.

Definitions.

164. Percentage is a part of a number expressed in *hundredths.*

The number on which percentage is computed is called the **base**; the number that denotes the percentage is called the **rate per cent.**, or simply the **rate**. Thus 15 per cent. of 80 *lbs.* is $\frac{15}{100}$ of 80 *lbs.*, or 12 *lbs.* Here, 80 *lbs.* is the *base*, 15 per cent. is the *rate*, and 12 *lbs.* is the *percentage.*

The **amount** is the base increased by the percentage, and the **difference** is the base diminished by the percentage. Thus, in the illustration just given, the *amount* is 80 *lbs.* + 12 *lbs.*, or 92 *lbs.*, and the *difference* is 80 *lbs.* — 12 *lbs.*, or 68 *lbs.*

Both the amount and difference are percentages of the base. Thus, 92 *lbs.* is 115 per cent. of 80 *lbs.*, and 68 *lbs.* is 85 per cent. of 80 *lbs.*

The term *per cent.* is usually denoted by the sign %.

Rate per cent. may be indicated in any of the ways shown in the following

TABLE OF EQUIVALENTS.

7	*per cent.*,	or	7 %	is equivalent to	$\frac{7}{100}$,	or to	.07;
10	“	“	10 %	“ “ “	$\frac{10}{100}$,	“ “	.1;
$12\frac{1}{2}$	“	“	$12\frac{1}{2}$%	“ “ “	$\frac{12\frac{1}{2}}{100}$,	“ “	.125;
100	“	“	100 %	“ “ “	$\frac{100}{100}$,	“ “	1;
125	“	“	125 %	“ “ “	$\frac{125}{100}$,	“ “	1.25;
	&c.,		&c.,	&c.,		&c.	

Principles of Percentage.

165. The **rate per cent.** is the ratio of the base to the percentage; hence, from Article **155**, we have the following principles:

1°. *The percentage is equal to the base multiplied by the rate.*

2°. *The base is equal to the percentage divided by the rate.*

3°. *The rate is equal to the percentage divided by the base.*

From the definition of the *amount* we have the following principles:

4°. *The amount is equal to the base multiplied by* 1 *plus the rate.*

5°. *The base is equal to the amount divided by* 1 *plus the rate.*

From the definition of the *difference* we have the following principles:

6°. *The difference is equal to the base multiplied by 1 minus the rate.*

7°. *The base is equal to the difference divided by 1 minus the rate.*

166. The following rules are based on the principles of Article **165**:

1°. *The base and rate being given, to find the percentage:*

RULE.

Multiply the base by the rate.

EXAMPLES.

1. What is 3% of \$650? *Ans.* \$19.50.
2. What is 9% of 711 *lbs.*? *Ans.* 63.99 *lbs.*
3. What is $3\frac{1}{4}\%$ of \$1,200? *Ans.* \$39.
4. What is 7% of 810 *yds.*? *Ans.* 56.7 *yds.*
5. Find $3\frac{3}{8}\%$ of 392 *miles.* *Ans.* 13.23 *mi.*
6. What is 6% of \$500? *Ans.* \$30.
7. Find 12% of $\$\frac{1}{12}$. *Ans.* 1 *ct.*
8. What is 11% of 31.25 *kilogrammes*? *Ans.* 3.4375 *kilog.*

2°. *The base and rate being given, to find the amount, or difference.*

RULE.

Multiply the base by 1 plus the rate to find the amount, and by 1 minus the rate to find the difference.

EXAMPLES.

9. If 694 *lbs.* is increased by 10% of itself, what is the amount? *Ans.* 763.4 *lbs.*

10. If $19.25 is diminished by 10% of itself, what is the difference? *Ans.* $17.325.

11. What is the amount of $329.62½, increased 20% of itself? *Ans.* $395.55.

12. If £2,971 is diminished by 37½% of itself, what is the difference? *Ans.* £1,856 17*s.* 6*d.*

The *amount* can be found by adding the percentage to the base, and the *difference*, by subtracting the percentage from the base. Let the following examples be solved in this manner.

13. A. has 600 eggs and B. has 15% more than A.; how many has B.? *Ans.* 690.

14. A farmer has 52 tons of hay and sells 16⅔% of it; how much has he left? *Ans.* 43 *T.* 6 *cwt.* 2⅔ *qrs.*

15. Of a farm containing 118 acres, 30% is arable and the remainder woodland; how many acres of each kind?
Ans. 35.4 acres of arable, and 82.6 of woodland.

16. Increase 4 *kilometers* by 20% of itself and find the amount. *Ans.* 4.8 *kilometers.*

17. Diminish 11 *hectoliters* by 3% of itself and find the difference. *Ans.* 10.67 *hectoliters.*

3°. *The percentage and rate being given, to find the base:*

RULE.

Divide the percentage by the rate.

EXAMPLES.

18. The percentage is \$50 and the rate 5%; what is the base? *Ans.* \$1,000.

19. The percentage is 10 *yds.* 3.05 *qrs.* and the rate 7%; what is the base? *Ans.* 153 *yds.* 3 *qrs.*

20. The percentage is \$220 and the rate is $33\frac{1}{3}$%; what is the base? *Ans.* \$660.

21. The percentage is $112\frac{1}{2}$ *gals.* and the rate $12\frac{1}{2}$%; what is the base? *Ans.* 900 *gals.*

22. The percentage is \$$460.31\frac{1}{4}$ and the rate 25%; what is the base? *Ans.* \$1,841.25.

23. The percentage is 9 *gals.* 3 *qts.* .07 *pts.* and the rate is 1%; what is the base?
Ans. 15 *hhds.* 30 *gals.* 3 *qts.* 1 *pt.*

4°. *The base and the percentage being given, to find the rate:*

RULE.

Divide the percentage by the base.

EXAMPLES.

24. The percentage is \$3 and the base \$50. What is the rate? *Ans.* 6%.

25. What per cent. of 16 is 7? *Ans.* $43\frac{3}{4}$%.

26. What per cent. of 25 is $\frac{2}{5}$? *Ans.* $1\frac{3}{5}$%.

27. What per cent. of 100 is $\frac{3}{8}$? *Ans.* $\frac{3}{8}$%.

28. What per cent. of $62\frac{1}{2}$ is $\frac{9}{10}$? *Ans.* $1\frac{11}{25}$%.

29. What per cent. of 300 is 60? *Ans.* 20%.

30. What per cent. of 64 is 250? *Ans.* $390\frac{5}{8}$%.

31. What per cent of £1 is 2*s.* 8*d.*? *Ans.* $13\frac{1}{3}$%.

32. What per cent. of £1 is 15*s.* 9*d.*? *Ans.* 78$\frac{3}{4}$%.

33. What per cent. of 1 mile is 140 *rds.*?

Ans. 43$\frac{3}{4}$%.

34. What per cent of 3.375 is 18.25? *Ans.* 540$\frac{20}{27}$%.

5°. *The amount and the rate being given, to find the base:*

RULE.

Divide the amount by 1 plus the rate.

EXAMPLES.

35. A number being increased by 12% of itself amounts to 336; what is the number? *Ans.* 300.

36. A. sold goods for $3,840, and in so doing cleared 20% on the original cost; what did they cost?

Ans. $3,200.

37. A dealer sold 40 sheep at an advance of 55 per cent. on what they cost him and received $248; what did they cost him? *Ans.* $160.

38. The population of a certain town in 1870 is 15,340, which is an increase of 18 per cent. on its population in 1860; what was the population in 1860?

Ans. 13,000.

39. A drover sold cows and sheep for $9,180; he received for his sheep 70% of what he got for his cows; what did he get for the cows? *Ans.* $5,400.

40. A farmer raises wheat and corn; his wheat crop is worth $1,036, which is 40% more than the value of his corn crop; what is the value of the corn crop?

Ans. $740.

6. *The difference and rate being given, to find the base:*

RULE.

Divide the difference by 1 minus the rate.

EXAMPLES.

41. A number being diminished by 30% of itself, the remainder is 385; what is the number? *Ans.* 550.

42. From a cask of wine 37% was drawn off and it was then found that 33.39 gallons remained; how many gallons did the cask contain? *Ans.* 53.

43. A. invests 35% of his capital in land and has $13,000 remaining; what is his capital? *Ans.* $20,000.

44. An army loses 27% of its number in battle and has 22,630 men remaining; how many did it contain originally? *Ans.* 31,000.

45. A. inherited a fortune, 37½% of which he spent and then had $31,250 remaining; how much did he inherit? *Ans.* $50,000.

46. B. purchased stocks and sold them at a loss of 15%, receiving for them $9,775; how much did he pay for them? *Ans.* $11,500.

MISCELLANEOUS EXAMPLES.

47. A person having a bill of $350 against another, consents to deduct 8% for cash; how much does he receive? *Ans.* $322.

48. A man had $5,420; he bought goods with 37½% of it, and then lent 25% of the balance to a friend; how much had he left? *Ans.* $2,540.62½.

49. A general had an army of 10,816 men, of whom he lost in action $6\frac{1}{4}\%$; how many did he lose?

Ans. 676.

50. A merchant bought 15 pieces of cloth, each containing $31\frac{1}{4}$ yards, and found on examination that 50% was damaged; how much was good? *Ans.* $234\frac{3}{8}$ *yds.*

EXAMINATION QUESTIONS.

(164.) What is percentage? The base? The rate? Illustrate. What is the amount? The difference? Illustrate. How is rate per cent. expressed? **(165.)** State the principles of percentage. **(166.)** Give the rule for finding percentage. How do you find the amount and difference? How do you find the base from the percentage and rate? From the amount or difference, and rate? How do you find the rate?

II. COMMISSION.

Definitions.

167. **Commission** is a percentage paid to an agent for transacting business.

An **agent** is one who transacts business for another; if he buys and sells merchandise, he is called a **commission merchant**; if he buys *stocks, bonds, real estate*, and the like, he is called a **broker**; if he collects *dues, taxes*, and the like, he is called a **collector.**

A quantity of merchandise sent to an agent for sale is called a **consignment**; the party that sends the goods is the **consignor**; and the agent who receives them is the **consignee.**

In case of merchandise, real estate, and the like, percentage is based on the sum received, or paid out.

The sum that remains after deducting the commission and other charges is called the **net proceeds.**

168. All the problems in commission can be solved by the rules for percentage, explained in Article **166**.

EXAMPLES.

1. A commission merchant received a consignment of flour, which he sold for $3,750; what was his commission at $4\frac{1}{2}\%$? *Ans.* $168.75.

2. A cotton broker sells 70 bales of cotton at $80 per bale; what is his commission at 3%? *Ans.* $168.

3. A real estate broker sells a house and lot for $23,750 and receives a commission of $1\frac{1}{8}\%$; how much must he pay to his principal? *Ans.* $$23,482.81\frac{1}{4}$.

4. A drover sells a drove of cattle for $4,250 and receives a commission of 4%; what does he pay to the owner of the cattle? *Ans.* $4,080.

5. A. sold 300 *bbls.* of flour at $7 per *bbl.*; what is his commission at $2\frac{1}{2}\%$? *Ans.* $52.50.

6. An auctioneer sold a house and furniture for $26,750; what was his commission at $1\frac{1}{8}\%$? *Ans.* $$300.93\frac{3}{4}$.

7. A commission merchant sold 500 pieces of bombazine for $30 a piece and paid to the owner $14,700; what rate of commission did he charge?

Ans. 2 per cent.

8. A commission merchant sold 500 pieces of muslin, each piece containing 21 yards, for 23 cents a yard; what was the amount of his commission at $2\frac{1}{2}\%$?

Ans. $$60.37\frac{1}{2}$.

Find the net proceeds of the following *accounts of sales :*

(9.)

Sales on account of

S. A. BETTS, *Romford, Ct.*

1873.	BUYER.	DESCRIPTION.	$	cts.
Jan. 4.	S. T. D.	78 *bu.* oats, @ 65*c.* . . .	50	70
" 15.	P. Q. R.	115 *bu.* corn, @ 93*c.* . . .	106	95
Feb. 11.	R. S. V.	1320 *lbs.* butter, @ 31*c.* . . .	409	20
March 5.	P. S. Q.	2560 *lbs.* cheese, @ 14*c.* . . .	358	40
		Gross amount	925	25
		Charges.		
		Freight and cartage $39.54		
		Storage 5.25		
		Commission on $925.25 @ 4½ % . . . 41.63⅝	86	42⅝
		Net proceeds	$838	82⅜

(10.)

Sales on account of

JOS. P. QUINN, *Roxbury, N. Y.*

1873.	BUYER.	DESCRIPTION.	$	cts.
June 13.	C. D.	1563 *lbs.* pork, @ 10*c.* . . .		
" "	D. L.	408 *lbs.* ham, @ 19*c.* . . .		
July 5.	S. K.	829 *lbs.* lard, @ 14*c.* . . .		
		Gross amount . . .		
		Charges.		
		Freight and cartage $5.42		
		Commission @ 5 % 17.49⅖		
		Net proceeds	$326	96⅗

11. A real estate broker purchased a house for $21,300, on which he was to receive a commission of $1\frac{1}{4}\%$; after a few days he was directed to sell the property again, which he did, obtaining an advance of 20% on the purchase price; allowing the same rate for selling as for buying, what was his total commission?

Ans. $585.75.

12. A commission merchant receives $3,825 to be invested in the purchase of flour, and charges 2% on what he pays out; what does he pay for the flour and what is his commission?

Here the amount is $3,825 and the rate is 2%; hence, the problem falls under the 4° rule for percentage, (Art. **166**); dividing $3,825 by 1.02 we find $3,750 for the cost of the flour, and consequently $75 for the commission.

13. A merchant in New York sends $12,600 to a commission merchant in Chicago to buy flour, agreeing to pay 5% on the cost of the flour for commission; how many barrels of flour does he receive, the market price being $12 per barrel? *Ans.* 1,000.

14. A manufacturer wishes to invest $22,050 in cotton; the market price is 15 cents a pound, the commission is $2\frac{1}{2}\%$, the insurance is $1\frac{1}{4}\%$, and the freight and cartage is $1\frac{1}{4}\%$; how many pounds can he buy?

Ans. 140,000.

15. A commission merchant sold 120 pieces of muslin, each piece containing $30\frac{1}{4}$ *yds.*, at 22 *cts.* per yard, on which he received a commission of $5\frac{1}{2}\%$; what did he receive? *Ans.* $43.923.

16. A butter dealer sold 75 firkins of butter, each weighing 56 *lbs.*, at 22½ cents a pound; what was his commission at 5%? *Ans.* $47.25.

17. An auctioneer sold 150 *hhds.* of sugar, each hogshead weighing 1,150 *lbs.*, at $7 per hundred, on which he charged a commission of 1¼%; what was his commission and what did he pay to the owner?

$$Ans. \begin{cases} \$150.93\tfrac{3}{4} \text{ commission.} \\ \$11,924.06\tfrac{1}{4} \text{ to owner.} \end{cases}$$

18. A broker receives $3,500 to invest in cotton at 8 cents per pound; if he charges 1¾% on the purchase, how many pounds of cotton does he purchase?

Ans. 42,997½ *lbs.*

19. A real estate broker bought a house, charging 2½% for his services; he sent in a bill for $3,224.24; what did he pay for the house? *Ans.* $3,145.60.

EXAMINATION QUESTIONS.

(167.) What is commission? An agent? A commission merchant? A broker? A collector? (168.) On what principles are the problems of commission solved?

III. INSURANCE.

Definitions.

169. A **policy of insurance** is a contract by which a company agrees to pay a certain sum of money to the party insured, in the event of loss, or accident.

The **premium** is a percentage paid to the company as a compensation for the risk assumed. The amount insured is the *base* of the percentage, and the *rate per cent.* is dependent on the degree of risk.

There are various branches of insurance; as, *Fire Insurance*, *Marine Insurance*, *Life Insurance*, *Accident Insurance*, and the like; these differ from each other in the nature and extent of the risk assumed and consequently in the mode of determining the *rate per cent.* The *rate per cent.* and the *times of paying the premium* having been settled, the method of proceeding is essentially the same in all branches.

170. The problems of insurance are solved by the rules for percentage explained in Article **166**.

EXAMPLES.

1. What premium must be paid to insure a house for \$8,000 for 1 year at $\frac{5}{8}\%$? *Ans.* \$50.

2. What is the premium for insuring a ship and cargo valued at \$73,850 at the rate of $3\frac{1}{4}\%$? *Ans.* \$2,400.12½.

3. A. insured his house for 1 year for \$8,000 at the rate of $\frac{1}{2}\%$, and his furniture for \$3,000 at the rate of $\frac{7}{8}\%$; what was the total premium? *Ans.* \$66.25.

4. A merchant insures his store for \$12,000 at the rate of $\frac{3}{4}\%$, and his stock of goods for \$15,000 at the rate of $1\frac{1}{4}\%$; what is the entire premium? *Ans.* \$277.50.

5. B. owns $\frac{3}{4}$ of a cargo of goods worth \$25,000, and insures his interest at the rate of $2\frac{1}{2}\%$; what is the premium? *Ans.* \$468.75.

6. A vessel and cargo valued at $37,900 are insured at the rate of 3%; what is the premium?

Ans. $1,137.

7. A shipping merchant sends two cargoes of wheat valued at $1,200, from Chicago to New York: he wishes to insure them at the rate of 1¾%; what premium must he pay? *Ans.* $21.

8. A man insures his house for $10,000 at ⅜%, his barn for $1,800 at ⅞%, and his furniture for $3,000 at 1¼%; what premium does he pay? *Ans.* $90.75.

9. A cargo of goods valued at $48,000, insured at 1½%, is injured to the amount of 37% of its value; what must the company pay, over and above the premium?

Ans. $17,040.

10. A merchant pays $2,340 on a vessel and cargo, the rate being 4½%; for what sum is he insured?

Ans. $52,000.

11. A man pays $87.50 for the insurance of his house at ⅞%, and $50 for the insurance of his furniture at 1¼%; if both are destroyed by fire how much will he receive? *Ans.* $14,000.

12. Shipped 5,000 barrels of flour worth $10.50 per barrel, and paid for insurance $2,887.50; what was the rate per cent? *Ans.* 5½ *per cent.*

13. A person 20 years of age is required to pay 1.38% per annum to insure his life; what must he pay each year on a policy of $5,000? *Ans.* $69.

14. A man 40 years of age wishes to insure his life for 5 years and finds that the annual rate is 1.86%; how much must he pay per year on a policy of $12,500?

Ans. $232.50.

15. To what amount must I insure a cargo of goods, at the rate of 2%, that I may, in case of loss, receive $50,000 over and above the premium?

Ans. $\frac{\$50000}{.98} = \$51,020.4082.$

16. What does it cost to insure a vessel for $13,000, at $2\frac{1}{8}\%$, and her cargo for $18,268.50, at $1\frac{7}{8}\%$?

Ans. $618.7844.

17. What does it cost to insure $\frac{3}{4}$ of a vessel, worth $27,500, at $3\frac{2}{5}\%$, and $\frac{3}{5}$ of the cargo, worth $126,875, at 2%?

Ans. $2,223.75.

18. A merchant shipped 400 *bbls.* of fish, worth $3.50 per barrel; for what amount must he insure, at 5%, to cover the value of the fish and the cost of insurance?

Ans. $1,473.6842.

19. I have a stock of goods worth $37,560.75, which I insure for $\frac{4}{5}$ of its value, paying $178.20; what is the rate?

Ans. $\frac{593}{1000}\%$.

20. A merchant insured 450 pieces of silk, each piece containing $35\frac{1}{4}$ yards, worth $1 per yard, at $4\frac{1}{2}\%$; what was the cost of insurance?

Ans. $\$713.81\frac{1}{4}$.

EXAMINATION QUESTIONS.

(169.) What is a policy of insurance? The premium? **(170.)** By what rules are problems in Insurance solved?

IV. PROFIT AND LOSS.

Definitions.

171. The *cost* of an article of merchandise is the sum paid for it; the *selling price* is the sum received for it.

If the selling price is *greater* than the cost, the difference is called *profit*; if the selling price is *less* than the cost, the difference is called *loss*.

It is customary in business transactions to regard profit, or loss, as a *percentage*, the cost being taken as the *base*.

172. The following examples of *profit* and *loss* can be solved by the rules for percentage explained in Article **166**.

EXAMPLES.

1. A merchant sold goods that cost $2,350 at an advance of 20%; what was his profit? *Ans.* $470.

2. A person entered into a speculation in which he invested $7,000, and cleared 15% on his investment; what was his profit? *Ans.* $1,050.

3. Bought a horse for $325, and sold him again at an advance of 18%; what was the profit? *Ans.* $58.50.

4. A merchant buys cloth at $6 per yard, and wishes to sell it again so as to clear 25%; what must he ask per yard? *Ans.* $7.50.

5. A person begins business with a capital of $18,000, and in one year increases it by 12%; what is his capital at the beginning of the second year? *Ans.* $20,160.

6. Bought a hogshead of sugar, weighing 1,280 *lbs.*, at $7\frac{1}{2}$ cents per *lb.*, and sold it at a profit of 25%; what was the entire profit? *Ans.* $24.

7. Bought drugs for £150, and sold them at a profit of 250%; what was the profit? *Ans.* £375.

8. A merchant had a stock of goods worth $3,750, of which $66\frac{2}{3}$ per cent. were destroyed by fire; what was his loss? *Ans.* $2,500.

9. Bought a house for $24,500, expended $1,500 in repairs, $4,000 for furniture, and $800 for taxes; what must the whole be sold for, to get an advance of 14%?

Ans. $35,112.

10. Bought a thousand tons of coal, at $5 per ton, and sold the same at a profit of 11%; what was the entire gain? *Ans.* $550.

11. Bought butter at 18 *cts.* a pound, and lost by the purchase 33⅓ per cent.; what was the selling price?

Ans. 12 *cts.*

12. Bought a ⅗ interest in a factory, and afterward sold ⅓ of my share for $15,000, making thereby 50% on the cost price; what was the value of the whole factory at the time of purchase? *Ans.* $50,000.

13. Sold goods at a loss of 8% on their cost, and received $8,280; what did they cost? *Ans.* $9,000.

14. Sold a horse for $364, gaining 12% on its cost; what did it cost? *Ans.* $325.

15. If I sell a horse for $240, and lose 20%, what should I have sold him for to gain 10%?

Ans. $330.

16. A person hires a piece of ground for $120 a year, and spends $625 for 50 Durham calves; in addition, he expends $250 for incidental expenses; how much must he receive per head for the calves at the end of the year to realize 20% on his expenditure? *Ans.* $23.88.

17. A stationer sold quills at $3.75 per thousand, by which he cleared 25% on their cost; how many per cent. would he have cleared, if he had sold them at $4.50 per thousand? *Ans.* 50%.

18. A man bought a factory for $8,000, and stocked it with material at a cost of $13,500; the building caught fire and was totally destroyed, but 60% of the material was saved; what was his total loss? *Ans.* $13,400.

19. A grocer bought 500 bags of coffee, each bag containing 49¼ *lbs.*, at 12 cents a pound, and sold it at a profit of 16⅔%; for what did he sell it?

Ans. $3,447.50.

20. A wool dealer bought 375 T. 15 *cwt.* of wool, at $75 per ton, and sold it again at a profit of 20%; what was his profit? *Ans.* $5,636.25.

21. A grocer bought 410 *bu.* of potatoes, at 96 *cts.* a bushel, and sold the lot for $492; what per cent. of profit did he make? *Ans.* 25%.

22. A merchant sold goods for $900, losing 10%; what did they cost him? *Ans.* $1,000.

23. A dealer sold 20 bbls. of flour for $7½ per barrel, and made 25%; what did all the flour cost him?

Ans. $120.

24. A person laid in 25 tons of coal during the summer, and saved thereby 25% on the winter price, which was $7.20 per ton; what did his coal cost him?

Ans. $135.

25. A merchant bought 250 bbls. of flour, at $5.50 *per bbl.*, and sold the lot for $1,875; how much per cent. did he make? *Ans.* $36\frac{4}{11}$%.

26. A grocer sold coffee at 40 cents a pound, and cleared 25%; for what should he have sold it to clear 12½%? *Ans.* 36 *cts.*

27. A machinist sold an engine for $24,000, and lost 4%; how much would he have received for it if he had made 10%? *Ans.* $27,500.

EXAMINATION QUESTIONS.

(171.) What is the cost of an article? The selling price? The profit? The loss? **(172.)** By what rules do we solve problems in profit and loss?

V. ASSESSMENT OF TAXES.

Definitions.

173. A *tax* is a sum of money paid by the inhabitants of a town, county, or state, for the support of government.

An *assessment* is a distribution of a tax according to fixed principles.

Property is of two kinds, *real* and *personal.* *Real estate* is fixed property, as, houses and lands; *personal property* is movable property, as, money, cattle, furniture, and the like.

Both real estate and personal property are liable to taxation in all the states; in some of the states every male citizen over 21 years of age is liable to a personal tax called a *poll-tax;* each person so taxed is called a *poll.*

174. In assessing taxes, the first thing to be done is to make a complete list of all the taxable property in the town, county, or state, with its assessed valuation, and also, when a poll-tax is levied, a list of all the polls.

The number of polls, multiplied by the tax on each poll, is then subtracted from the entire tax, and the remainder is the tax on property.

Having found the tax on property, we divide it by the assessed value of all the property, both real and personal, and the quotient is the *rate* of the tax.

To find the tax on any individual, we have the following

RULE.

Multiply the number of polls for which he is liable by the tax on each poll, and to the result add the product of the assessed value of his property by the rate of the tax.

EXAMPLES.

1. A town is to be taxed $4,280; the assessed value of the property, both personal and real, is $1,400,000, and there are in the town 390 polls, each liable to a tax of $2; what is the rate of taxation? *Ans.* $\frac{1}{4}$%.

2. In the town just mentioned, A. is liable for 4 poll-taxes and is assessed for $3,750; what is his entire tax? *Ans.* 17.37\frac{1}{2}$.

3. In the same town, B. is liable for 2 polls and is assessed for $8,290; what is his tax? *Ans.* 24.72\frac{1}{2}$.

4. In a school district, a tax of $375 is levied for the support of schools; what is A.'s tax on a valuation of $8,000, the entire valuation of the district being $120,000? *Ans.* $25.

5. What is the tax on a valuation of $9,875, the rate being 1%? *Ans.* $98.75.

EXAMINATION QUESTIONS.

(173.) What is a tax? An assessment? Define real estate. Personal property. (174.) Explain the method of assessing taxes, in detail.

VI. SIMPLE INTEREST.

175. Interest is a percentage paid for the use of money. It is reckoned at a certain rate per cent. for each year.

The sum on which interest is paid is called the **principal**; the per cent. per annum is called the **rate**; and the sum of the principal and interest for a given time, is called the **amount.**

The rate of interest is fixed by law, but it differs in the several states. Any interest greater than that allowed by law, is called **usury.**

If interest is computed on the principal only, it is called **simple interest**; if computed on the principal and also on the accrued interest as it falls due, it is called **compound interest.**

Interest depends on the *principal*, the *rate*, and the *time.*

In reckoning time, *a month* is usually regarded as $\frac{1}{12}$ *of a year*, without reference to the number of days it may contain, and *a day* is regarded as $\frac{1}{30}$ *of a month.*

176. Problems in *simple interest* may be solved by the rules for percentage given in Article **166**, but on account of their importance, separate rules will be given for each case.

1°. *The principal, the rate, and the time in years being given, to find the interest:*

RULE.

Multiply the principal by the rate, and that result by the time in years.

EXAMPLES.

1. What is the interest on $860 for 3 years at 6%?

Explanation.—The interest on $860 for 1 year is equal to $860 multiplied by .06, that is, to $51.60; hence, the interest for 3 years is 3 times as great, or $154.80.

OPERATION.

$$\begin{array}{r} \$860 \\ .06 \\ \hline \$51.60 \\ 3 \\ \hline \$154.80 \end{array}$$

In all the examples in simple and compound interest, let the decimals be carried to four places, adding 1 to the last figure retained when the following figure is equal to or greater than 5.

2. What is the interest on $794 for 3½ years at the rate of 7%? *Ans.* $194.53.

3. Find the interest on $8,942 for 3¼ years at 8½%. *Ans.* $2,470.2275.

4. Compute the interest on $8,720 for 1½ years at 7%. *Ans.* $915.60.

5. Find the interest on $712 for 3 years at 6%. *Ans.* $128.16.

6. What is the interest on $329.50 for 2 years at 7%? *Ans.* $46.13.

7. What is the interest on $986.30 for 1 year at 6½%? *Ans.* $64.1095.

8. Find the interest on $12,600 for 1 year at 4⅖%. *Ans.* $554.40.

To find the amount, add the interest to the principal:

9. To what will $1,400 amount in 2 years at the rate of 3½%? *Ans.* $1,498.

10. Find the amount of \$4,186.25 for 1 year at $5\frac{1}{2}\%$.
Ans. \$4,416.4937.

11. What is the amount of £168 for 1 year at 7%?
Ans. £179 15*s.* 2*d.* $1\frac{3}{5}$ *far.*

12. What is the amount of \$1,001.75 for 1 year at $6\frac{5}{8}\%$?
Ans. \$1,068.1159.

13. What does \$450 amount to in $2\frac{1}{2}$ years at 6%?
Ans. \$517.50.

14. Find the amount of \$3,875.20 for 5 years at $4\frac{3}{4}\%$.
Ans. \$4,795.56.

15. What does £2,000 amount to in 7 years at $8\frac{1}{3}\%$?
Ans. £3,166 13*s.* 4*d.*

16. To what will \$7,500 amount in 5 years at the rate of $3\frac{1}{4}\%$?
Ans. \$8,718.75.

2°. *The principal, the rate, and the time in months being given, to find the interest:*

RULE.

Multiply the principal by the rate and divide the product by 12; then multiply the quotient by the number of months.

17. What is the interest on \$480 for 9 months at the rate of 7%?

OPERATION.
\$480
.07
12)\$33.60
\$2.80
9
Ans. \$25.20

Explanation.—The interest on \$480 for 1 year is \$33.60; hence, the interest on the same sum for 1 month is $\frac{1}{12}$ of \$33.60, or \$2.80; consequently, the interest for 9 months is 9 times as great, or \$25.20.

18. What is the interest on \$815 for 11 months at 7%?
Ans. \$52.2958.

19. What is the interest on $1,375 for 4 months at 5%? *Ans.* **$22.9167.**

20. What is the interest on $1,742.10 for 7 months at 5½%? *Ans.* **$55.8924.**

21. Find the interest on $840 for 9 months at 4½%. *Ans.* **$28.35.**

22. What is the interest on $711 for 14 months at 8%? *Ans.* **$66.36.**

23. What is the amount of $1,285 for 16 months at 6%? *Ans.* **$1,387.80.**

24. What is the interest on $748 for 8 months at 6%? *Ans.* **$29.92.**

In many of the States the legal rate of interest is 6%, which is equal to ½% per month. In this case we multiply the principal by one-half the number of months, and divide the result by 100, or, what is the same thing, we divide half the number of months by 100, and multiply the principal by the quotient.

25. What is the interest on $180 for 8 months, at 6%?

Explanation.—Here we multiply the principal by .04, which is the same thing as multiplying by half the number of months and dividing by 100.

OPERATION.

$$\begin{array}{r} \$180 \\ .04 \\ \hline \textit{Ans.}\ \$19.20 \end{array}$$

26. What is the interest of $890 for 10 months, at 6%? *Ans.* **$44.50.**

27. Find the interest on $1,175 for 14 months, at 6%. *Ans.* **$82.25.**

28. What is the interest on $8,742.75 for 9 months, at 6%? *Ans.* **$393.42⅜.**

29. Find the interest on $846 for 15 months, at 6%.
Ans. $63.45.

30. What is the interest on $750 for 14½ months, at 6%?
Ans. $54.37½.

31. Find the amount of $872 for 10 months, at 6%.
Ans. $915.60.

If the time is given in *years*, *months*, and *days*, reduce it to months and decimals of a month, and proceed as before.

32. What is the interest on $480 for 3 years, 4 months, and 21 days, at the rate of 7%?

Explanation.—We find the interest for 1 month, as explained before; this is $2.80. We then reduce the time to months and decimals of a month, which gives 40.7 months. Multiplying the interest for 1 month by 40.7, we have $113.96 for the required interest.

OPERATION.

```
      $480
         7
12)$33.60
     $2.80
      40.7
Ans. $113.96
```

33. What is the interest on $1,640 for 4 years, 5 months, and 12 days, at 7%? *Ans.* $510.86.

34. What is the interest on $2,306 for 1 year, 7 months, 27 days, at 5%? *Ans.* $191.20.

35. Find the interest on $1,260 for 3 years and 6 days, at 7%. *Ans.* $266.07

36. What is the interest on $1,620 for 5 *yrs.* 24 *da.*, at 4%? *Ans.* $328.32.

37. What is the interest on $675.89 for 3 *yrs.* 6 *mos.* 6 *da.*, at 8%? *Ans.* $190.1506.

38. What is the interest on $864.768 for 9 *mos.* 25 *da.*, at 6¼%? *Ans.* $44.2892.

39. What is the amount of $3565.20 for 3 *yrs.* 9 *mos.*, at 10½%? *Ans.* $4968.9975.

40. What is the interest on $100 for 1 *yr.* 3 *mos.* 10 *da.*, at 6%? *Ans.* $7.6667.

41. What is the interest on $1,000 for 9 *mos.* 15 *da.*, at 7%? *Ans.* $55.4167.

42. What is the interest on $1,700 for 2 *yrs.* 3 *mos.* 10 *da.*, at 5%? *Ans.* $193.6102.

43. What is the interest on £2,500 for 5 *mos.* 18 *da.*, at 4½%? *Ans.* £52 10*s.*

44. What is the interest on $450 for 3 *yrs.* 6 *mos.* 18 *da.*, at 5%? *Ans.* $79.875.

45. What is the interest on $710 for 3 *yrs.* 10 *mos.*, at 7%? *Ans.* $190.5166.

46. What is the interest on $1,766 for 1 *yr.* 4 *mos.* 18 *da.*, at 6%?

The rate being 6%, we divide half the number of months by 100, which gives .083, and multiply the principal by the quotient; this gives $1,766 × .083 = $146.578. *Ans.*

47. What is the interest on $14.50 for 19 days, at 6%? *Ans.* $14.50 × .3167 = $0.0459.

48. What is the amount of £10,000 for 3 *yrs.* 7 *mos.* 12 *da.*, at 6%? *Ans.* £12,170.

49. What is the interest of $2,300 from May 3d, 1870, to January 15th, 1873, at 6%? *Ans.* $372.60.

50. What is the amount of $3,150 from August 16th, 1861, to May 1st, 1869, at 6%? *Ans.* $4606.875.

51. Find the amount of $5,675 for 3 *yrs.* 9 *mos.* 24 *da.*, at 6%. *Ans.* $6974.57½.

52. What is the interest on $3,000 for 4 *yrs.* 8 *mos.* 6 *da.*, at 6%? *Ans.* $843.

53. What is the interest on $2,500 for 1 *yr.* 8 *mos.* 12 *da.*, at 7%? *Ans.* $297.50.

54. What is the amount of $3,500 for 2 *yrs.* 4 *mos.* 18 *da.*, at 4½%? *Ans.* $3,875.37½.

55. What is the amount of $850 for 1 *yr.* 9 *mos.* 15 *da.*, at 7½%? *Ans.* $964.2188.

56. What is the interest on $1,800 for 2 *yrs.* 3 *mos.* 10 *da.*, at 8%? *Ans.* $328.

57. What is the interest on $2,100 for 9 *mos.* 12 *da.*, at 4½%? *Ans.* $74.025.

58. What is the interest on $1,100 for 11 *mos.* 6 *da.*, at 7%? *Ans.* $71.8667.

59. What is the interest on $3,000 for 3 *yrs.* 6 *mos.* 18 *da.*, at 5%? *Ans.* $532.50.

60. What is the amount of $4,000 for 1 *yr.* 3 *mos.* 12 *da.*, at 4%? *Ans.* $4,205.3333.

61. What is the amount of $5,000 for 1 *yr.* 1 *mo.* 6 *da.*, at 7%? *Ans.* $5,385.

Interest for Days at 6%.

In business transactions, interest for days is computed on the supposition that 30 days make 1 month, and 12 months 1 year, that is, that the year consists of 360 days. In this case, when the rate is 6%, the interest on $1 for 6 days is 1 *mill;* hence, the following

RULE.

Multiply the principal by the number of days, divide the result by 6, and then move the decimal point three places to the left.

62. What is the interest on \$84.60 for 15 days, at 6%?

Ans. $\frac{\$84.60 \times 15}{6000} = \$.2115.$

63. What is the interest on \$175.20 for 18 days, at 6%?

Ans. \$.5256.

64. What is the amount of \$144 for 25 days, at 6%?

Ans. \$144.60.

Having found the interest for the given time at 6%, we may find it for other rates by the method of aliquot parts.

65. What is the interest on \$960 for 24 days, at 5% and at 7%?

The interest at 6% is \$3.84; if we diminish this by its sixth part, we find the interest at 5% equal to \$3.84 — \$.64, or \$3.20; if we increase it by its sixth part, we find the interest at 7% equal to \$3.84 + \$.64 = \$4.48.

66. What is the interest on \$1,230 for 84 days, at 5% and at 7%? *Ans.* \$14.35 and \$20.09.

67. What is the interest on \$960 for 66 days at 4½% and at 7½%?

The interest at 6% is \$10.56; diminishing and increasing this by its fourth part, we have the interest at 4½%, \$7.92, and at 7½% we have \$13.20.

68. What is the interest on \$648 for 54 days at 4½% and 7½%? *Ans.* \$4.374 and \$7.29.

Accurate Interest for Days.

The preceding method gives a result too great by its $\frac{1}{73}$ part; to find the accurate interest, we may diminish the result found in accordance with the preceding rule by its $\frac{1}{73}$ part, or we may use the following

RULE.

Find the interest for 1 year at the given rate; then multiply the result by the number of days and divide the product by 365.

69. What is the accurate interest on $803 for 35 days at 7%?

$$\text{\textit{Ans.}}\ \frac{\$56.21 \times 35}{365} = \$5.39.$$

70. What is the accurate interest on $584 for 70 days at 7%? *Ans.* $7.84.

71. What is the accurate interest on $876 for 105 days at 8%? *Ans.* $20.16.

3°. *The principal, the interest, and the time being given, to find the rate:*

RULE.

Find the interest at 1% on the principal for the given time; then divide the given interest by the result.

72. The interest on $500 for 1 *yr.* 6 *mos.* is $45; what is the rate?

Explanation.—The interest on $500 for $1\frac{1}{2}$ years at 1% is $7.50. But the total interest on $500 for the same time is $45. Hence, if we divide $45 by $7.50, the quotient, 6, will be the rate per cent.

OPERATION.

$$\$500 \times .01 \times 1.5 = \$7.50$$

$$\frac{\$45}{\$7.50} = 6\%.$$

73. The interest on \$950 for 16 *mos.* is $\$88.66\frac{2}{3}$; what is the rate per cent? *Ans.* 7%.

74. The interest on \$380 for 1 *yr.* 4 *mos.* is \$22.80; what is the rate? *Ans.* $4\frac{1}{2}$%.

4°. *The principal, the rate, and the interest being given, to find the time:*

RULE.

Find the interest on the principal for 1 year at the given rate; then divide the given interest by the result.

75. The interest of \$1,200 at 6% is \$120; how long is it at interest?

Explanation.—The interest on \$1,200 for 1 year is \$72. Hence, it will require $1\frac{2}{3}$ years for the same sum to give \$120 interest; but $1\frac{2}{3}$ years is equal to 1 *yr.* 8 *mos.*

OPERATION.

$$\$1200 \times .06 = \$72.$$

$$\frac{\$120}{\$72} = 1\frac{2}{3}\ yrs. = 1\ yr.\ 8\ mos.$$

76. The interest on \$880 at 5% is \$66; how long does it remain at interest? *Ans.* 1 *yr.* 6 *mos.*

5°. *The interest, the rate, and the time being given, to find the principal:*

RULE.

Find the interest on \$1 for the given time at the given rate; then divide the given interest by the result.

77. The interest on a given sum for 20 *mos.* at 6% is \$65; what is the principal?

Explanation.—The interest on \$1 for 20 *mos.* is 10 *cts.*; now if \$1 draws 10 *cts.* in the given time, it will require \$650 to draw \$65 in the same time; hence, $\frac{\$65}{10\ cts.,}$ is equal to the number of dollars in the required principal.

OPERATION.

$$\$1 \times .06 \times 1\tfrac{2}{3} = \$0.10$$

$$\frac{\$65}{\$0.1} = \$650.$$

$\therefore$ \$650. *Ans.*

78. The interest on a given sum at 5% for 18 *mos.* is \$157.50; what is the principal? *Ans.* \$2,100.

79. The interest on a given sum for 2 *yrs.* 6 *mos.* at 6% is \$450; what is the sum? *Ans.* \$3,000.

80. The interest on a certain principal for 3 *yrs.* 4 *mos.* at 4½% is \$412.50; what is the principal? *Ans.* \$2,750.

Notes.

177. A **promissory note** is a written promise to pay a sum of money, either on demand, or at some specified time.

The person who signs the note is called the *maker*, and the party that has legal possession of it is called the *holder*.

A **negotiable note** is one that is payable either to *order*, or to *bearer*.

Form of a Negotiable Note.

\$375.

NEW YORK, *October* 16, 1873.

For value received I promise to pay to John Doe, or order, three hundred and seventy-five dollars on demand, with interest at 7 per cent.

RICHARD ROE.

In this case John Doe, the person named in the note, is called the *payee;* he can make the note transferable by writing his name across the back; he is then called an *indorser*, and is obliged to pay the money when it falls due, if the maker of the note, Richard Roe, fails to do so.

The **face** of a note, or bill, is the sum named in it. Thus, in the note just given, the face is $375.

Partial Payments.

178. A **partial payment** is a payment of a part of the amount due on a note.

The date and the amount of each partial payment is *indorsed*, that is, *written on the back of the note*, and is to be taken into account in making the settlement.

The following method of settling a note, or other interest-bearing obligation, has been sanctioned by the Supreme Court of the United States, and is now adopted in New York, Massachusetts, and many other States:

SUPREME COURT RULE.

I. Find the amount of the given principal up to the time when the sum of the partial payments is equal to, or exceeds, the interest then due: from this result subtract the sum of the partial payments to the time considered.

II. Take the remainder for a new principal and proceed as before, continuing the operation to the time of final settlement.

EXAMPLES.

1. On a note dated May 1, 1866, for $1,200 at 6%, are the following indorsements:

Nov. 1, 1866, received $100.
Mar. 1, 1867, " $20.
Sept. 1, 1868, " $180.

What was due on the note Nov. 1, 1869?

OPERATION.

Given principal	$1200
Int. to Nov. 1, 1866 (6 *mos.*)	$36
Amount	$1236
1st payment	$100
1st new principal	$1136
Int. to Sept. 1, 1868 (22 *mos.*)	$124.96
Amount	$1260.96
Sum of 2d and 3d payments	$200
Second new principal	$1060.96
Int. to Nov. 1, 1869 (14 *mos*)	$74.2672
Amount due Nov. 1, 1869	$1135.2272

Explanation.—We compute the interest on $1.200 from the date of the note to the time of the first payment, and since the first payment is greater than the interest then due, we add the interest to the principal and subtract the first payment, which gives $1,136 for a new principal. We then see, by inspection, that the interest on the new principal from Nov. 1, 1866, to Mar. 1, 1867 (4 *mos.*), is greater than the second payment; we therefore compute the interest on the new principal to the time of the third payment, add it to the principal, and from the result subtract the sum of the 2d and 3d payments, which gives another new principal. We then find the amount of this principal up to the time of settlement.

2. On a note dated May 1, 1866, for $350 at 6%, are the following indorsements:

Dec. 25, 1866, received $50.
Sept. 1, 1868, " $20.
June 13, 1869, " $100.

What was due April 13, 1870? *Ans.* $252.0735.

3. On a note dated July 1, 1869, for $700 at 7%, are the following indorsements.

July 1, 1870, received $200.
July 1, 1871, " $400.

What is due July 1, 1872? *Ans.* $200.5501.

4. On a note dated May 1, 1860, for $2,000 at 7%, are the following indorsements:

May 1, 1861, received $500.
May 1, 1862, " $450.
May 1, 1863, " $750.
May 1, 1864, " $400.

What was due May 1, 1865? *Ans.* $311.7605.

5. On a note dated Aug. 1, 1870, for $1,000 at 5%, are the following indorsements:

Dec. 1, 1870, received $310.25.
April 1, 1871, " $225.50.
Aug. 1, 1872, " $400.00.

What was due Jan. 1, 1873? *Ans.* $128.1514.

If partial payments are made on obligations due within a single year, the balance is usually adjusted by the following rule, called the

MERCANTILE RULE.

Find the amount of the principal from the date of the note to the time of settlement; find the amount of each payment from the time it was made to the time of settlement, and subtract their sum from the first result.

In applying this rule, the times are reduced to days, and the interest is then computed by the rule for days.

6. On a note dated Jan. 1, 1873, for $1,000 at 7%, were the following indorsements:

Feb. 15, 1873, received $200.
May 16, 1873, " $400.

What was due Aug. 14, 1873?

OPERATION.

Amt. of $1000 for 225 *da.*		$1043.75
Amt. of $200 for 180 *da.* . . .	$207	
Amt. of $400 for 90 *da.* . . .	$407	
Sum of amts. of payments		$614
Balance due Aug. 14, 1873 . .		$429.75

7. On a note dated Jan. 1, 1873, for $800 at 6%, are the following indorsements:

Feb. 6, 1873, received $200.
April 30, 1873, " $210.

What is due on the note June 5, 1873? *Ans.* $405.44.

8. On a note dated July 1, 1872, for $1,870 at 6%, was the following indorsement:

Dec. 4, received $840.

What was due Feb. 2, 1873? *Ans.* $1,088.92.

EXAMINATION QUESTIONS.

(175.) What is interest? What is the principal? The rate? The amount? Define simple interest. Compound interest. On what does interest depend? How is time reckoned? **(176.)** Give the rule for finding interest when the time is expressed in years. The amount. Give the rule when the time is expressed in months, or in years, months, and days. Give the rule for days at 6%. At any other %. Give the rule for accurate interest for days. How do you find the rate, from the principal, interest, and time? How do you find the time, from the principal, rate, and interest? How do you find the principal, from the interest, rate, and time? **(177.)** What is a promissory note? Who is the maker? The holder? What is a negotiable note? Give its form. **(178.)** What is a partial payment? Give the Supreme Court rule for computing interest on partial payments. The mercantile rule.

VII. COMPOUND INTEREST.

179. **Compound Interest** is interest computed on the principal and also on the accrued interest as it falls due.

Interest may be added to principal at the end of each *year*, *half year*, or other *fixed period*. Unless otherwise stated, the period is supposed to be a year. In savings banks, and similar institutions, the period is generally a half year.

180. From the principles already explained we have the following

RULE FOR COMPOUND INTEREST.

I. Find the amount of the given principal for the first period; then find the amount of this result for the second period; and so on to the *end of the given time; the final result will be the total amount.*

II. From the total amount subtract the given principal and the remainder will be the compound interest.

EXAMPLES.

1. What is the compound interest on $642 for 2 years at 6% per annum, interest being compounded annually?

OPERATION.

$642 . . . Principal,
1.06
$680.52 . . . 1st amount,
1.06
$721.3512 . . . Total amount,
$642
$79.3512 . . . Compound int.

Explanation. — Here the period is 1 year, and the amount of $1 for that period is $1.06; multiplying $642 by 1.06, we find the amt. for the first period, $680.52; multiplying this by 1.06, we find its amount for the second period, $721.3512. Subtracting the original principal, we find $79.3512 for the required interest.

2. What is the compound interest on $918 for 3 years at 6% per annum, interest being compounded annually?
Ans. $175.3527.

3. What is the *amount* of $650 for 4 years at 6% per annum, interest being compounded semi-annually?

Here the period is one half of a year, and the amount of $1 for each period is $1.03. Proceeding as before, we find the amount. *Ans.* $823.4003.

The operation of computing the amount of any sum for a given time may be shortened by the use of the following

TABLE,

Showing the amount of $1 at compound interest, for any number of periods from 1 to 20.

PERIODS.	2%.	2½%.	3%.	3½%.	4%.	5%.	6%.	7%.
1	1.0200	1.0250	1.0300	1.0350	1.0400	1.0500	1.0600	1.0700
2	1.0404	1.0506	1.0609	1.0712	1.0816	1.1025	1.1236	1.1449
3	1.0612	1.0769	1.0927	1.1087	1.1249	1.1576	1.1910	1.2250
4	1.0824	1.1038	1.1255	1.1475	1.1699	1.2155	1.2625	1.3108
5	1.1041	1.1314	1.1593	1.1877	1.2167	1.2763	1.3382	1.4026
6	1.1262	1.1597	1.1941	1.2293	1.2653	1.3401	1.4185	1.5007
7	1.1487	1.1887	1.2299	1.2723	1.3159	1.4071	1.5036	1.6058
8	1.1717	1.2184	1.2668	1.3168	1.3686	1.4775	1.5938	1.7182
9	1.1951	1.2489	1.3048	1.3629	1.4233	1.5513	1.6895	1.8385
10	1.2190	1.2801	1.3439	1.4106	1.4802	1.6289	1.7908	1.9672
11	1.2434	1.3121	1.3842	1.4600	1.5395	1.7103	1.8983	2.1049
12	1.2682	1.3449	1.4258	1.5111	1.6010	1.7959	2.0122	2.2522
13	1.2936	1.3785	1.4685	1.5640	1.6651	1.8856	2.1329	2.4098
14	1.3195	1.4130	1.5126	1.6187	1.7317	1.9799	2.2609	2.5785
15	1.3459	1.4483	1.5580	1.6753	1.8009	2.0789	2.3966	2.7590
16	1.3728	1.4845	1.6047	1.7340	1.8730	2.1829	2.5404	2.9522
17	1.4002	1.5216	1.6528	1.7947	1.9479	2.2920	2.6928	3.1588
18	1.4282	1.5597	1.7024	1.8575	2.0258	2.4066	2.8543	3.3799
19	1.4568	1.5987	1.7535	1.9225	2.1068	2.5270	2.0256	3.6165
20	1.4859	1.6386	1.8061	1.9898	2.1911	2.6533	2.2071	3.8697

4. What is the amount of $820 for 6 years at the rate of 4% per annum, interest being compounded semi-annually?

In this case there are 12 periods and the rate for each period is 2%. From the table, we find that $1 amounts to $1.2682 in that time, at the given rate, and because $820 amounts to 820 times as much as $1, we have,

$1.2682 × 820 = $1,039.924. *Ans.*

5. What is the amount of $900 for 9 years at 7% per annum, interest being compounded semi-annually?

Ans. $1.8575 × 900 = $1,671.75.

6. What is the compound interest on $1,850 for 3 years at 8% per annum, interest being compounded quarterly? *Ans.* $1.2682 × 1,850 — $1,850 = $496.17.

7. Find the amount of $800 for 14 years at 7% per annum, interest being compounded annually.

Ans. $2,062.80.

If the last period is fractional, compute the amount to the end of the next preceding period, and then find the amount of that result for the fractional period.

8. What is the amount of $500 for 3 years 2 months, at 6% per annum, interest being compounded annually?

The amount for 3 years is $1.191 × 500 or $595.50, and this in two months amounts to $601.455. *Ans.*

9. What is the amount of $1,200 for 4 *yrs.* 8 *mos.* at 7%, interest being compounded annually?

Ans. $1,646.3648.

10. What is the amount of $1,350 for 5 *yrs.* 4 *mos.* at 6%, interest being compounded semi-annually?

Ans. $1,850.5503.

EXAMINATION QUESTIONS.

(179.) What is compound interest? How often may interest be added to principal? **(180.)** What is the rule for compound interest? How is the table used?

VIII. Discount.

181. Discount is a percentage deducted from the face of a bill, debt, or note.

Commercial Discount.

182. Commercial Discount is a percentage to be deducted from the face of a bill of merchandise.

The face of the bill is the *base* and the difference between this and the discount is called the *net proceeds*.

From definitions and preceding principles we have the following

RULE.

I. Multiply the face of the bill by the rate per cent. and the product will be the discount.

II. Subtract the discount from the face of the bill and the difference will be the net proceeds.

EXAMPLES.

1. What is the discount on a bill of $350 at 5%, and what is the net proceeds?

Ans. The discount is $17.50 and the net proceeds, $332.50.

2. Sold a bill of merchandise amounting to $1,173, deducting 10% for cash; what was the net proceeds?

Ans. $1,055.70.

3. Flour is sold on credit at $12.50 per barrel; what is the cash price, the discount being 15%?

Ans. $10.62½.

4. Find the discount on a bill of goods whose face is $1,200, at the rate of 2½%. *Ans.* $30.

5. Sold a lot of goods amounting to $918, deducting 12½% for cash; what was the net proceeds?

Ans. $803.25.

6. Sold 56 firkins of butter, each weighing 42¼ *lbs.*, at 22 cents a pound, deducting 5% for cash; what was the net proceeds? *Ans.* $\$494.49\frac{4}{10}$.

7. Sold 50 *bbls.* flour at $7.50 per barrel, deducting 7½% for cash; what was the net proceeds?

Ans. $346.87½.

Present Value and True Discount.

183. The present value of a debt is a sum, which placed at interest, will amount to the face of the debt when it becomes due.

The **true discount** is the difference between the face of the debt and its present value.

The **rate of discount** is the rate per annum multiplied by the time in years.

In this case the face of the debt is the *amount*, and the present value is the *base*: hence, we have the following

RULE.

I. Divide the face of the debt by 1 plus the rate of discount, and the quotient will be the present value.

II. Subtract the present value from the face of the debt and the remainder will be the true discount.

EXAMPLES.

1. What is the present value of a debt of $1,500 due 1 *yr.* and 4 *mos.* hence, money being worth 6% per annum? *Ans.* $1,388.8889.

In this example the rate per annum is .06 and the number of years is $1\frac{1}{3}$; hence, the rate of discount is .08; dividing $1,500 by 1.08 we find the answer.

2. What is the true discount on $1,200, due 2 years hence, money being worth 7%?

$$Ans.\ \$1{,}200 - \frac{\$1{,}200}{1.14} = \$147.37.$$

3. What is the present value of a debt of $1,760, due 3 *yrs.* and 6 *mos.* hence, at the rate of 6% per annum? *Ans.* $1,454.5455.

4. Find the true discount on a debt of $1,141.25, due 7 *mos.* 15 *days* hence, at 6% per annum. *Ans.* $41.25.

5. What is the true discount on $730, due in 2 years, at 5% per annum? *Ans.* $66.3636.

6. Find the present value of a debt of $986, due 2 *yrs.* and 8 *mos.* hence, at 6% per annum. *Ans.* $850.

7. What is the true discount in the last example? *Ans.* $136.

8. What is the present value of $1,200, due in 1 *yr.* 4 *mos.*, at $7\frac{1}{2}\%$ per annum? *Ans.* $1,090.9091.

9. A debt of $1,400 is due in 9 months; what is the true discount, interest being computed at the rate of 6%? *Ans.* $60.2871.

10. Find the present value of a note for $750, due in 1 *yr.* 8 *mos.* 12 *da.*, at 6%. *Ans.* $680.5808.

11. What is the present value of a note for $1,300, due in 2 *yrs.* 8 *mos.*, at 7%? *Ans.* $1,095.5056.

12. What is the present value of $10,000, due in 4 *mos.* 18 *da.*, at 4½%? *Ans.* $9,830.4252.

Banks and Bank Discount.

184. A **Bank** is a corporation for receiving deposits, loaning money, and issuing bills to circulate as currency.

Interest on money lent is computed *in advance* and deducted from the face of the note; this operation is called *discounting* and the percentage deducted is called *bank discount*. The difference between the face of the note and the bank discount is called the *proceeds* of the note.

A note is said to *mature* when it becomes *legally* due; this is *three* days after it is *nominally* due. The three days that elapse after a note is nominally due, before it becomes legally due, are called *days of grace*. If the last day of grace falls on Sunday or on a legal holiday the note is legally due, in some of the States on the preceding day and in some on the following day. The days of grace are included in the time for which interest is computed.

Two kinds of notes are discounted at bank, *business notes*, or business paper, and *accommodation notes*, or accommodation paper.

A *business note* is a note given from one party to another in the ordinary course of business.

An *accommodation note* is a note made for the purpose of borrowing money.

A business note offered for discount may be due at some future time, without interest, or it may be a note bearing interest, and due at a future time.

In the latter case, the *amount of the note at maturity* is taken as the *face* of the note, and the discount is computed on this amount.

EXAMPLES.

Find the *time of maturity*, the *bank discount*, and the *proceeds* of the following notes:

(1.)

$750.

NEW YORK, *Nov.* 10, 1873.

Sixty days after date, for value received, I promise to pay to John Doe, or order, seven hundred and fifty dollars, with interest at 7%. RICHARD ROE.

Discounted Nov. 10, 1873.

Here the note is nominally due on the 9th of January, 1874, but it is not legally due till the 12th of the same month; these facts are expressed by writing both dates as follows: Due Jan. $^{9}/_{12}$, 1874. The term of discount is 63 days and the interest on $750 for this time is $$9.18\frac{3}{4}$. Subtracting this from the face of the note, we find for the remainder $$740.81\frac{1}{4}$. Hence the

Ans. Due Jan. $^{9}/_{12}$, 1874; disc., $$9.18\frac{3}{4}$; proceeds, $$740.81\frac{1}{4}$.

(2.)

$1,250.

NEW YORK, *Sept.* 29, 1873.

Ninety days after date, we jointly and severally agree to pay to Samuel Poe, or order, twelve hundred and fifty dollars, with interest at 7%.

RICHARD ROE,
JOHN DOE.

Discounted Sept. 29, 1873.

Ans. Due Dec. $^{28}/_{31}$, 1873; discount, $22.6042;
proceeds, $1,227.3958.

(3.)

$1,680.

NEW YORK, *Sept.* 5, 1873.

Ninety days after date, for value received, I promise to pay to John Doe, or order, sixteen hundred and eighty dollars, with interest at 6%.

RICHARD ROE.

Discounted at 7%, Oct. 5, 1873.

Here the amount of the note at maturity is $1,706.04, and the discount on this for 63 days at 7% is $20.90; hence,

Ans. Due Dec $^{4}/_{7}$, 1873; discount, $20.90; proceeds, $1,685.14.

4. What is the bank discount on a note for $350, payable 90 days after date at 7%? *Ans.* $6.3292.

5. What is the proceeds of a note of $1,000, payable at bank, 60 days after date, at 6%? *Ans.* $989.50.

6. A. has a note against B. for $1,728, payable 90 days after date, which he gets discounted at the rate of 7%; what does he receive? *Ans.* $1,696.752.

To find the face of a note payable at a future time which will yield a given sum at a given rate:

RULE.

Divide the given sum by the proceeds of $1 for the given time and rate.

7. Find the face of a note payable 90 days after date which will yield $500, the rate being 6%.

The proceeds of $1 for 93 days at 6% is $.9845; dividing $500 by this result, we find $507.872. *Ans.*

8. A note having 90 days to run at the rate of 7% is to be made, so that the proceeds shall be $2,050; what is the face of the note? *Ans.* $2,087.7465.

EXAMINATION QUESTIONS.

(181.) What is discount? (182.) What is commercial discount? Rule? (183.) What is the present value of a debt? What is true discount? What is the rate of discount? What is the rule for present value and true discount? (184.) What is a bank? What is discounting? What is bank discount? When does a note mature? What are days of grace? What are business notes? Accommodation notes? Explain the method of computing discount on a note.

IX. STOCKS AND BONDS.

Definitions.

185. A **business corporation** is a company of persons authorized, under certain restrictions, to transact business as an individual.

The money used in carrying on the business of the corporation is the *capital stock*. This is divided into equal parts called *shares*.

The owners of the shares are called *shareholders* or *stockholders*; they control the business of the association.

The **par value** of a stock is the same as its nominal value. This is usually $100 per share, though it may be $50, $25, or any other sum.

The **market value** of a stock is the price it will bring in open market. If the market value is greater than the par value, the stock is said to be at a *premium*, or *above par;* if the market value is less than the par value, it is said to be at a *discount*, or *below par*.

Dividends are percentages paid to stockholders as profits on the business; **assessments** are percentages that stockholders are called on to pay to meet losses and extraordinary expenses.

The prices at which stocks are reported are *percentages* on their par value; thus, when a stock is quoted at 87, its market value is 87% of its par value.

186. A **bond** is a properly authenticated obligation to pay a sum of money, at or before a certain time.

Bonds are of two kinds: *government bonds*, for whose payment the public faith is pledged, and *bonds of incorporated companies*, for whose payment the property of the company is pledged.

The face of each bond is usually $1,000, but it may be any other sum.

Bonds are bought and sold like stocks, and the prices at which they are quoted are *percentages* on their par value.

In the United States, the term *stocks* includes all shares of incorporated companies, and all bonds for whose payment the public faith is pledged, such as, *national, state*, and *city bonds*.

187. Problems relating to stocks and bonds are solved by the principles of percentage and interest.

In what follows, the par value of each share of stock is supposed to be $100, the par value of each bond $1,000, and the brokerage, which is paid by the party for whom the purchase, or sale is made, is supposed to be $\frac{1}{4}$%.

EXAMPLES.

1°. *Applications of the rules for percentage*, (Art. **166**).

1. What is the market value of 120 shares of steamship company stock @ $53\frac{3}{4}\%$?

Ans. $\$12,000 \times .5375 = \$6,450$.

2. What is the cost, including brokerage, of 44 shares of railroad stock @ $129\frac{1}{2}\%$?

Ans. $\$4,400 \times 1.2975 = \$5,709$.

3. Sold \$5,000 in gold @ $112\frac{1}{4}\%$; what was the net proceeds after deducting brokerage? *Ans.* \$5,600.

4. A. sold 160 shares railroad stock @ $92\frac{3}{4}\%$ and purchased with the proceeds bank stock @ $73\frac{3}{4}\%$, paying brokerage on both sale and purchase; how many shares did he receive? *Ans.* $\frac{160 \times 92\frac{1}{2}}{74} = 200$.

5. What is the premium on 88 shares of bank stock @ 114%? *Ans.* $\$8,800 \times .14 = \$1,232$.

6. What is the discount on 190 shares of railroad stock @ 89%? *Ans.* \$2,090.

7. A. sold 93 United States bonds @ $113\frac{1}{2}\%$, paying brokerage on the sale; what did he receive?

Ans. \$105,322.50.

8. A speculator buys 225 shares of Erie stock @ $30\frac{1}{4}\%$ and sells it again @ $31\frac{1}{2}\%$, paying brokerage on both transactions; what does he gain? *Ans.* \$168.75.

9. If 170 railroad bonds cost, (including brokerage), \$192,100, what is their market rate? *Ans.* $112\frac{3}{4}\%$.

10. How much gold can be bought for $8,930 in currency, when gold is quoted @ $111\frac{5}{8}\%$, brokerage not considered? *Ans.* $8,000.

11. A broker sells 30 shares of bank stock @ $96\frac{1}{2}\%$, and 120 shares of railroad stock at 105%, retaining the usual brokerage; how much does he pay to his principal? *Ans.* $15,457.50.

2°. *Applications of the rules for interest,* (Art. **176**).

12. If I buy a 6% stock @ 90%, what rate of interest do I receive on the investment?

Here the *principal*, $90, yields an interest of $6 per annum; hence, (Case 3°, Art. **165**), we have,

$$\frac{\$6}{\$0.90} = 6\tfrac{2}{3}\%. \quad \textit{Ans.}$$

13. The market rate of a 5% stock is $85\frac{1}{2}\%$; if the purchaser pays brokerage, what rate of interest does he receive on his investment? *Ans.* $\frac{\$5}{\$.85625} = 5\frac{2875}{3425}\%$.

14. An 8% stock sells @ 112%; what rate of interest does it yield to the purchaser? *Ans.* $7\frac{1}{7}\%$.

15. At what rate must an 8% stock be purchased to yield the purchaser 7% interest?

In this case the price of one share is a principal which, at the rate of 7%, yields $8.00 in one year; hence, (Case 5°, Art. **165**), we have,

$$\frac{\$8.00}{\$.07} = 114\tfrac{2}{7}\%. \quad \textit{Ans.}$$

16. I wish to purchase a 5% stock on such terms as will give me 7% on my investment; how much can I pay for the stock, including brokerage? *Ans.* $71\frac{3}{7}\%$.

17. Which is the better purchase, a 5% stock @ 75%, or a 6% stock at 90%, brokerage not being considered?

Ans. Each yields $6\frac{2}{3}$% on the investment.

EXAMINATION QUESTIONS.

(185.) What is a business corporation? What is capital stock? Shares? Shareholders? Par value? Market value? Dividends? Assessments? (186.) What is a bond? (187.) By what rules do we solve problems relating to stocks and bonds?

X. EXCHANGE.

188. **Exchange** is a method of making payments at distant places by means of drafts, or bills of exchange.

This method of making exchanges may be illustrated by the following simple case: A flour merchant of Chicago forwards $5,000 worth of flour to a shipper in New York, and at the same time a dry goods merchant of Chicago buys $5,000 worth of merchandise from a New York importer. The flour merchant draws his draft for $5,000 on the shipper and sells it to the dry goods merchant; the latter forwards it to the importer, who presents it to the shipper and receives the money called for. In this way the debts in both cities are liquidated without the necessity of sending any money from either.

Dealers in exchange usually buy drafts on distant places and send them forward as a basis of credit; they then sell their own drafts drawn against this credit in sums to suit their customers.

189. A **draft or bill of exchange** is a written order from one party to another to pay to a third party a certain sum of money at a specified time.

There are always three parties to a bill of exchange and sometimes more.

The party that draws the bill is called the *drawer*, or *maker*; the party on whom it is drawn is called the *drawee;* and the party to whom the money is to be paid is called the *payee.*

The party that buys a bill of exchange is called a *buyer,* or *remitter;* if the payee, or any other party, writes his name across the back of the bill he is called an *indorser;* and the party that has legal possession of the bill at the time of payment is called the *holder.*

When the bill is presented to the drawee, if he agrees to pay it, he writes the word *accepted* and *signs his name* across the face of the bill; he is then called the *acceptor,* and the bill is said to be accepted.

If both drawer and drawee reside in the same country, the bill is called a *domestic,* or an *inland bill;* if they reside in different countries, it is called a *foreign bill.*

Par of Exchange.

190. The **par of exchange** between two places is the relative value of the principal unit of currency at the two places. Thus, £1 *sterling* is equal in value to $4.8665, and this is the par of exchange between London and New York; $1 is equal in value to 5.18 francs, and this is the par of exchange between New York and Paris.

If $4.8665 in New York will just purchase a bill of £1 on London, exchange on London is at par; if it will buy a larger bill, exchange is against London and in favor of New York; if it requires more than $4.8665 to buy a bill of £1, exchange is in favor of London and against New York.

The course of exchange is the variation in the price at one place for bills on another place. It depends on the relative demand and supply. If trade between New York and London is just balanced, exchange on London is at *par;* if New York owes London, the demand for drafts on London is greater than the supply and exchange on London is at a *premium;* if London owes New York, the demand for drafts on London is less than the supply, and exchange on London is at a *discount.*

Current exchange is the rate at which exchange is actually computed. Thus, if at any place the real exchange on New York is ½% and if the depreciation of the currency there is 2%, the current exchange is 2½%.

A **sight draft** is one that is payable on presentation; a **time draft** is one that is payable at a specified time, or at a certain time after presentation. In the latter case three days of grace are allowed, but not in the former.

Domestic or Inland Exchange.

191. All problems relating to inland exchange can be solved by the rules for percentage and bank discount. In applying the rules for percentage, the *face of the bill* is the *base*, the *premium*, or *discount* is the *rate per cent.*, and the cost of the bill is the *amount.*

EXAMPLES.

1. What is the cost of a sight draft for $800, exchange being at 1½% premium?

Ans. $800 × 1.015 = $812.

2. What is the cost of a sight draft on New York for $1,250, exchange being 1¼% discount?

Ans. $1,250 × .9875 = $1,234.37½.

3. What is the market price of a sight draft on New York for $890, exchange being worth 101¼%?

Here the rate of premium is 1¼%, and 1 plus the rate, that is, the amount per cent. is 1.0125; hence, the draft is worth $890 × 1.0125 = $901.12½. *Ans.*

4. Find the market value of a sight draft on New York for $1,800, exchange being 99%.

Ans. $1,800 × .99 = $1,782.

Both exchange and bank discount are computed on the face of a time draft. They may be computed separately, or together, as may be more convenient.

5. Find the cost of a draft on New York for $1,400, payable 60 days after sight, exchange being worth 102½% and interest being reckoned at the rate of 7% per annum.

OPERATION.

Amount of $1400 at 2½% premium . .	$1435.00
Interest on $1400 for 63 days at 7%. .	17.15
Difference	$1417.85 *Ans.*

Otherwise:

Per cent. premium	.025
Per cent. discount.	.01225
Difference	.01275

Hence, the cost is equal to $1400 × 1.01275

= $1417.85. *Ans.*

6. What is the cost of a draft on New York for \$2,400, payable 90 days after sight, interest being 10% and exchange 103%? *Ans.* \$2,410.

7. Find the market value of a draft on New York for \$1,650, payable 60 days after sight, exchange being 98½% and interest 6% per annum.

Ans. $\$1,650 \times .9745 = \$1,607.925$.

8. If exchange is 101½%, how large a sight draft can be bought for \$7,900? *Ans.* $\frac{\$7,900}{1.015} = \$7,783.2512$.

9. What is the face of a sight bill that can be bought for \$5,000, when exchange is 98½%?

Ans. \$5,076.1421.

10. What is the face of a draft payable 60 days after sight that will cost \$1,000, exchange being 3% premium and interest 6% per annum?

Ans. $\frac{\$1000}{1.03 - .0105} = \$980.873$.

Foreign Exchange.

192. Exchange between the United States and foreign countries is computed by the method of equivalents. The equivalent is made known by the current quotation.

It is customary to express the face of a bill in terms of the money of the place where it is payable.

Problems in exchange between the United States and foreign countries may be solved by the following

RULE.

Convert the face of the bill into dollars in accordance with the current quotation.

EXAMPLES.

1. What is the cost of a bill of £87, when £1 is worth $4.82? *Ans.* $419.34.

2. What is the price of a sight bill for £750 when £1 costs $4.85? *Ans.* $3,637.50.

3. What is the cost of a bill on London for £300, when £1 is worth $4.86? *Ans.* $1,458.

4. Find the cost of a bill on London for £315, when £1 is worth $4.80. *Ans.* $1,512.

5. How large a bill on London can be bought for $2,000 when exchange is quoted at $4.85?

Ans. £412 7*s.* 5*d.*

6. What is the cost of a sight bill on Paris for 5,000 francs, exchange being 5.15 francs to the dollar.

Ans. $\frac{5000}{5.15} = \$970.8738.$

7. What is the cost of a draft on Paris for 30,000 francs, when exchange is at the rate of $1 to 5.25 francs?

Ans. $5,714.2857.

8. How large a bill can be purchased on Paris for $1,000, when $1 is worth 5.20 francs?

Ans. $1{,}000 \times 5.20 = 5{,}200$ francs.

EXAMINATION QUESTIONS.

(188.) What is exchange? Illustrate its operation. **(189.)** What is a draft, or bill of exchange? What is the drawer? The drawee? The payee? The buyer, or remitter? An indorser? The holder? The acceptor? What is a domestic, or inland bill? What is a foreign bill? **(190.)** What is the par of exchange? Illustrate. What is the course of exchange? On what does it depend? What is current exchange? Illustrate. What is a sight draft? A time draft? **(191.)** By what rules do we solve problems of inland exchange? **(192.)** How do we solve problems of foreign exchange?

XI. EQUATION OF PAYMENTS.

193. Equation of Payments is the operation of finding the time at which the aggregate of several debts due at different times may be paid without loss of interest to either party.

The time at which the total payment may be made is called the *equated time*, and the corresponding date is called the *equated date.*

The time that a debt has to run is called its *time of credit*, and the date from which this time is reckoned is called the *initial date.*

A credit of $1 for a unit of time is called a *unit of credit* ; the product of a debt by its time of credit is its *amount of credit.*

The unit of time may be 1 day, or 1 month, but it must always be the same throughout the same problem.

Solutions of problems in *equation of payments* depend on the following principle:

The amount of credit of the aggregate of several debts is equal to the aggregate of the amounts of credit of the debts taken separately.

194. There may be three cases: 1°, the debts may be on one side, and have the same initial date; 2°, the debts may be on one side, and have different initial dates; 3°, some of the debts may be on one side, and some on the other, the dates of payment being different.

1°. *When the debts are on one side and have the same initial date.*

On the 1st of January, 1874, A. owes B. $400 payable in 9 months, $800 payable in 6 months, and $600 payable in 4 months; what is the equated time and at what date may the whole debt be paid without loss to either party?

OPERATION.

$$\begin{array}{rcccr} 400 & \times & 9 & = & 3600 \\ 800 & \times & 6 & = & 4800 \\ \underline{600} & \times & 4 & = & \underline{2400} \\ 1800\,) & & & & 10800\,(\,6 \end{array}$$

Ans. { Equated time = 6 months.
Equated date, July 1, 1874.

Explanation. — In this case the unit of credit is $1 for 1 month. Multiplying the different sums by their terms of credit and adding the products, we find the aggregate of the amounts of credit equal to 10,800 units; but this is equal to the amount of credit of $1,800 for the required time, (Art. 193); hence, 10,800 divided by 1,800 will give the number of months in the equated time. Adding this to the initial date we find the equated date.

Since all similar cases may be treated in the same manner, we have the following

RULE.

I. Multiply each debt by its time of credit and find the sum of the products; divide this by the sum of the debts and the quotient will be the equated time.

II. Add the equated time to the initial date and the result will be the equated date.

EXAMPLES.

1. A merchant owes $2,400, of which $400 is payable in 6 *mos.*, $800 in 10 *mos.*, and $1,200 in 16 *mos.*; what is the equated time? *Ans.* 12⅓ *months.*

2. A. owes B. $2,400, of which $800 is payable in 6 months, $600 in 8 months, and $1,000 in 12 months; what is the equated time? *Ans.* 9 *months.*

3. A merchant owes $300, payable as follows: $80 in 22 days, $100 in 60 days, and $120 in 75 days; what is the equated time? *Ans.* 56 *days.*

The rule gives 55$\frac{26}{30}$ days. In such cases we shall neglect the fraction when it is less than ½ and add 1 to the whole number when it is equal to, or greater than ½. If the unit of time is 1 month, the fractional part will be reduced to days at the rate of 30 days per month, and fractional parts of the latter will be treated as just explained.

4. A. owes B. $1,000, payable as follows: $200 at present, $400 at 6 months and $400 at 15 months; what is the equated time? *Ans.* 8 *months* 12 *days.*

5. A merchant owes a certain sum of money, of which ⅙ is payable at 30 days, ⅓ at 60 days, and the remainder at 90 days; what is the equated time for the payment of the whole? *Ans.* 70 *days.*

2°. *When the debts are on one side, and have different initial dates.*

6. A. sells goods to B. on a credit of 60 days, as follows:

Jan. 14, 1874, a bill of $2,000;
Feb. 10, " " " " $1,500;
March 25, " " " " $3,000;

Required the equated time and date of payment.

Explanation.—The first payment is due March 15, the second April 11, and the third May 24. Assuming the date of the earliest payment as the initial date, the other payments will have 27 and 70 days to run. Proceeding as before, we find the equated time greater than 38.5 days; calling it 39 days, according to the rule, and adding it to the assumed initial date, we find the equated date to be March 15 + 39 *da.* = April 23.

OPERATION.

March 15,	2000 × 0 =	00000
April 11,	1500 × 27 =	40500
May 24,	3000 × 70 =	210000
	6500)	250500(38.538

Ans. { Equated time, 39 *days.*
Equated date, April 23, 1874.

Since all similar cases may be treated in the same manner, we have the following

RULE.

Find the date of each payment, and assume the earliest as an initial date; then find the time of credit of each item, and proceed as in the last case.

EXAMPLES.

7. A. bought goods as follows: May 5, $500 on 4 months' credit; May 25, $750 on 4 months; June 27, $800 on 6 months; what is the equated date of payment of the whole debt? *Ans.* Oct. 26.

When credit is given by months, calendar months are understood, without reference to the number of days they contain; in applying the preceding rule, the times of credit, if not even months, are reduced to days, counting the actual number of days in each month. Thus, in the example just given, the items mature Sept. 5, Sept. 25, and Dec. 27, and the times of credit, counting from Sept. 5 as the initial date, are 0, 20, and 113 days.

8. There are three notes payable as follows: the *first* for $500 on the 12th of February, 1853, the *second* for $400 on the 12th of March, and the *third* for $300 on

the 1st of April; what is the equated date for the payment of them all? *Ans.* March 5, 1853.

3°. *When each party owes the other, the times of payment being different.*

A statement of the items of *debit* and *credit* between two parties is called an *account;* and the difference between the sum of the items on the *debit* and *credit* sides of the account, is called the *balance of account.*

By preceding rules, all the items on either side may be reduced to a single item, payable at a specified time. Hence, every account may be reduced to two items, one on each side.

The following suppositions illustrate every case that can arise:

1°. Suppose that A. owes B. $500 payable July 16, 1874, and that B. owes A. $200 payable July 1, 1874.

2°. Suppose that A. owes B. $200 payable July 16, 1874, and that B. owes A. $500 payable July 1, 1874.

In both cases it is required to find the date at which the *balance of account* may be paid without loss to either party.

In both examples let the date of the earlier payment, July 1, 1874, be taken as the initial date.

In the first example we have the following

OPERATION.

$$500 \times 15 = 7500$$

$$\frac{7500}{300} = 25.$$

Ans. July 26, 1874.

Explanation.—On the 1st of July A. was entitled to 500 × 15, or 7,500, units of credit; but at that time the balance due B. was but $300; hence, the time before the balance became due was $\frac{7500}{300}$, or 25 days; adding this to the initial date, we find that the balance was due July 26, 1874.

In the second case we have the following

Explanation.—On the 1st of July A. was entitled to 200 × 15, or 3,000 units of credit; but B. was also entitled to the same amount; hence, the balance due him, $300, was entitled to credit for $\frac{3000}{300}$, or 10 days *before* the 1st of July, that is, from the 21st of June, 1874.

OPERATION.

$$200 \times 15 = 3000$$

$$\frac{3000}{300} = 10.$$

Ans. June 21, 1874.

Since all similar cases can be treated in the same manner, we have the following

RULE.

I. Reduce each side to a single item and take the date of the earlier payment as an initial date; then multiply the side of the account that falls due last by the time between the dates of payment and divide the product by the balance of the account; the quotient will be the equated time.

II. If the greater side of the account falls due last, add the equated time to the initial date; if the smaller side falls due last, subtract the equated time from the initial date; the result will be the equated date at which the balance is payable.

EXAMPLES.

9. A. owes B. $8,750 payable July 21, 1873, and B. owes A. $6,500 payable June 9, 1873; when and to whom is the balance payable?

Ans. It is payable to B. Nov. 19, 1873.

10. A. owes B. $6,500 payable July 21, 1873, and B. owes A. $8,750 payable June 9, 1873; to whom is the balance payable and what is its equated date?

Ans. To A., and the equated date is Feb. 8, 1873.

Equation of Accounts.

195. An **account** is a written statement of the items of debit and credit between two parties.

If the sums of the items on the two sides of an account are equal the account is said to be *balanced*.

The **balance** of an account is the difference between the sum of the debits and the sum of the credits. The balance is to be added to the less sum.

Equation of Accounts is the operation of finding the time at which the balance of an account should be made payable in order that there may be no loss of interest to either party. This time is called the *equated time*.

196. Let it be required to find the equated time of payment of the balance of the following account:

Dr. HENRY AHL *in acct. with* BENJ. BARROL. *Cr.*

1873.			1873.			
April 1.	Merchandise.	$375 00	April 20.	Cash . . .	$500	00
" 18.	"	250 00	May 20.	" . . .	185	00
May 25.	"	150 00				

The first side of the account is equivalent to a single item of $775 payable April 17 and the second side is equivalent to $685 payable April 28, (Case 2°, Art. **194**).

By the rule of Case 3°, Art. **194**, we find that the balance is payable 84 days preceding April 17, that is, on the 23d of January; hence the equated date of the balance is January 23, 1873.

All similar cases may be treated in like manner; hence, the following

RULE.

Equate each side separately by the rule for Case 2°, Art. **194**; *then find the equated time of the balance by the rule for Case 3°, Art.* **194.**

EXAMPLE.

Find the equated date of payment of the balance in the following account:

Dr. A. *in acct. with* B. *Cr.*

1874.				1874.			
May 1.	To corn . .	$400	00	June 1.	By cash . .	$300	00
June 15.	" wheat . .	1800	00	July 1.	" " . .	1500	00
Oct. 15.	" corn . .	1200	00	Nov. 1.	" " . .	1000	00
		$3400	00			$2800	00

Ans. April 25, 1874.

Cash Balance.

197. The **cash balance** of an account is the difference between the two sides of an account with interest on each item to the date of settlement.

The **interest balance** is the difference between the total interest on the items of the two sides.

From these definitions we have the following

RULE.

Add to each side of the account the total interest on all the items of that side to the date of settlement; the difference between the footings of the sides so increased is the cash balance.

EXAMPLES.

1. Find the cash balance of the following account on the 15th of August, 1874, interest being 6%:

Dr. JNO. IRVING *in acc't. with* HENRY HOUR. *Cr.*

1874.		$	*cts.*	1874.		$	*cts.*
April 1.	To mdse. .	25	00	May 10.	By cash . .	30	00
" 20.	" " . .	18	00	Aug. 12.	" " . .	35	00
June 15.	" " . .	40	00	" 15.	Cash bal. .	18	8218
Aug. 15.	Bal. of Int. .		8218				
		$83	8218			$83	8218

Explanation.—The debit side of the account is entitled to interest on $25 for 136 days, on $18 for 117 days, and on $40 for 61 days, all of which is equivalent to the interest on $1 for 7,946 days; in like manner, the credit side is entitled to interest on $1 for 3,015 days. The debit side is therefore entitled to a balance of interest equal to that on $1 for 4,931 days, or to $.8218. We add this to the debit side and then find that the sum of the items on that side exceeds the sum of the items on the credit side by $18.818. This is the cash balance and is to be added to the credit side.

The interest on $1 for 4,931 days is found by the rule for days, that is, we divide 4,931 by 6,000 and the quotient is the number of dollars in the required interest.

2. Find the cash balance of the following account on

the 20th of March, 1873, interest being computed at the rate of 7%:

Dr. C. *in acct. with* D. *Cr.*

1873.				1873.			
Feb. 5.	To mdse. . .	$680	00	Jan. 25.	To cash . .	$250	00
" 24.	" " . .	300	00	Feb. 15.	" " . .	600	00
Mar. 1.	" " . .	150	00				
" 16.	" " . .	600	00				

Ans. $881.6314.

EXAMINATION QUESTIONS.

(193.) What is the equation of payments? Equated time? Equated date? Time of credit? Initial date? Define a unit of credit. Amount of credit. On what principle does the equation of payments depend? **(194.)** What three cases may arise? Give the rule in each case. **(195.)** What is an account? A balance? What is the equation of accounts? **(196.)** What is the rule for equating an account? **(197.)** What is a cash balance? Give the rule for finding it.

XII. CUSTOM-HOUSE BUSINESS.

198. Duties are taxes laid by the government on goods imported into the United States.

The ports at which foreign goods may be landed are called *ports of entry.* At each port of entry is a *custom house,* at which the goods liable to pay duties are *appraised* and the taxes *collected.*

Duties are of two kinds, *specific* and *ad valorem.* A **specific duty** is a definite tax laid on a certain article without reference to its cost. An **ad valorem duty** is a percentage on the cost of an article at the place from which it is imported.

An **invoice** or **manifest** is an inventory or list of goods setting forth their cost price at the place from which they are imported.

A **tonnage duty** is a tax laid on a vessel for the privilege of entering a port. This tax depends on the size, or tonnage, of the vessel, the ton being equal to a capacity of 40 cubic feet.

In computing specific duties certain allowances are made for losses incidental to commerce.

The weights of heavy articles are expressed in tons of 2,240 *lbs.* The weight of an article before any allowance is made is called its *gross weight*; its weight, after deducting all allowances, is called its *net weight.*

Draft is an allowance for general waste; it is deducted before any other allowance is made.

Tare is an allowance for the weight of the box, or case, containing goods.

Leakage is an allowance for waste of liquids imported in casks.

Breakage is an allowance for waste of liquids imported in bottles.

EXAMPLES.

1. What is the net weight of 176 *hhds.* of sugar, each weighing 10 *cwt.* 1 *qr.* 14 *lbs.*, draft being 7 *lbs.* per *cwt.* and tare 56 *lbs.* per *hhd.*? *Ans.* 1,623 *cwt.* 3 *qrs.* 14 *lbs.*

In computing the weight, it is to be remembered that 28 *lbs.* make 1 *qr.*, 4 *qrs.* 1 *cwt.* and 20 *cwt.* 1 *ton.* In this case the gross weight is 1,826 *cwt.*; the draft is 114 *cwt.* 14 *lbs.*; and the tare is 88 *cwt.*

2. What is the net weight of 100 tierces of rice, each weighing 250 *lbs.*, tare being 5%? *Ans.* 23,750 *lbs.*

3. A merchant imported goods invoiced as follows:

300 *yds.* cloth @ 13*s.* per *yd.*;
873 *yds.* carpeting @ 4*s.* per *yd.*;
150 *doz.* cambric hdkfs. @ 2*s.* 8*d.* per dozen.

The duty on the cloth was 30%; on the carpeting 25%; and on the hdkfs. 33%; what was the amount of duty in dollars, allowing $4.8665 to the pound sterling?

Ans. $529.2319.

4. A wine merchant imported wines as follows:

75 baskets champagne @ $12 per basket;
18 casks Madeira @ $45 per cask;
12 casks sherry @ $35 per cask.

Allowing 4% for leakage on the wine in casks, what is the amount of duty @ 45%? *Ans.* $936.36.

5. What is the duty on 54 *T.* 13 *cwt.* 3 *qrs.* 20 *lbs.* of iron, invoiced at $45 per ton, the duty being 33⅓%?

Ans. $820.446.

EXAMINATION QUESTIONS.

(198.) What are duties? Define specific and ad valorem duties. What is an invoice? What is tonnage duty? What is gross weight? Net weight? Draft? Tare? Leakage? Breakage?

CHAPTER VII.

POWERS, ROOTS, PROGRESSIONS, AND ANNUITIES.

I. POWERS.

Definitions.

199. A **Power** is the product of two or more equal factors. One of these factors is called the *root* of the power.

The product of two equal factors is called a *second power*, or *square*; the product of three equal factors is a *third power*, or *cube*; the product of four equal factors is a *fourth power*; and so on. Thus, 3 × 3, or 9, is the square of 3; 3 × 3 × 3, or 27, is the cube of 3, and so on.

The number itself, though not strictly a power, is sometimes called the *first power*. Thus, in naming the different powers of 3, we say 3 is the *first* power, 9 is the *second* power, 27 is the *third* power, and so on.

The **exponent** of a power is a number that shows how many times the root is taken as a factor. It is written to the right and above the root. Thus, the expression 3^4 indicates that the root 3 is to be taken four times as a factor. In this case, the exponent is 4 and the expression 3^4 is equivalent to $3 \times 3 \times 3 \times 3$, or 81.

Operation of Finding any Power.

200. From the preceding definitions, it is obvious that we may find any power of a number by the following

RULE.

Take the number as a factor as many times as there are units in the exponent of the power.

The operation of finding the powers of numbers is called *involution*, or *raising to powers*.

EXAMPLES.

Raise the following numbers to the powers indicated:

1. 121^2.	*Ans.* 14641.
2. 247^2.	*Ans.* 61009.
3. 615^2.	*Ans.* 378225.
4. 973^2.	*Ans.* 946729.
5. 227^3.	*Ans.* 11697083.
6. 951^3.	*Ans.* 860085351.

EXAMINATION QUESTIONS.

(**199.**) What is a power? Define a second power. A third power. A fourth power. What is an exponent? (**200.**) What is the rule for finding a power?

II. Roots.

Definition.

201. A **root** of a number is one of its equal factors.

A *square root* is one of two equal factors; thus, 5 is the square root of 25.

If a number can be resolved into two equal factors, it is said to be a *perfect* square; if it cannot be so resolved it is not a *perfect square*.

All the perfect squares less than 100, with their square roots, are written in the following

TABLE.

Perfect squares,	1	4	9	16	25	36	49	64	81;
Square roots,	1	2	3	4	5	6	7	8	9.

A *cube root* is one of three equal factors; thus, 5 is the cube root of 125.

If a number can be resolved into three equal factors, it is said to be a *perfect* cube; if it cannot be so resolved it is an *imperfect* cube.

All the perfect cubes less than 1,000, with their cube roots, are written in the following

TABLE.

Perfect cubes,	1	8	27	64	125	216	343	512	729;
Cube roots,	1	2	3	4	5	6	7	8	9

The *extraction* of a root of a number is the operation of finding one of its equal factors.

1°. Square Root.

202. The demonstration of the rule for extracting the square root of a number depends on the principles of algebra, (Manual of Algebra, Art. **107**). In accordance with those principles we have the following

RULE.

I. Separate the given number into periods of two figures each, beginning at the right hand: the period on the left will often contain but one figure.

II. Find the greatest perfect square in the first period on the left and place its square root on *the right, after the manner of a quotient in division; subtract the square of this root from the first period, and to the remainder bring down the second period for a dividend.*

III. Double the root found and place it on the left for a divisor. See how many times this divisor is contained in the dividend, exclusive of the right hand figure, and place the quotient in the root and also at the right of the divisor.

IV. Multiply the divisor, thus augmented, by the last figure of the root already found, subtract the product from the dividend and to the remainder bring down the next period for a new dividend.

V. Double the root already found for a new divisor, and continue as before, until all the periods have been brought down and operated on.

If the last remainder is 0, the given number is a perfect square and the root is exact; if not, the root is true to within less than 1.

If any quotient figure proves too large, let it be diminished until it gives a product less than the partial dividend.

EXAMPLES.

1. Find the square root of 8836.

OPERATION.

```
      88̇36̇(94
      81
     ----
18،4)73،6
      73 6
      ----
         0
```

Explanation.—The two periods are 88 and 36; the greatest perfect square in 88 is 81, (table, Art. 201), and its square root is 9; this we write as the first figure of the root and place its square 81 under the first period; subtracting, we have 7 for a remainder, to which we bring down the period 36 for a dividend; doubling 9 we have 18, which we place on the left for a divisor, and this is contained 4 times in 73; we therefore place 4 on the right of 9 and also on the right of 18; multiplying 184 by 4 we find 736, which taken from 736 gives 0 for a remainder; hence, the square root of 8836 is 94.

The square root of a number is indicated by the sign $\sqrt{\ }$; thus, $\sqrt{49} = 7$.

Perform the following indicated operations:

2. $\sqrt{9604}$.	*Ans.* 98.
3. $\sqrt{13225}$.	*Ans.* 115.
4. $\sqrt{342225}$.	*Ans.* 585.
5. $\sqrt{944784}$.	*Ans.* 972.
6. $\sqrt{1104601}$.	*Ans.* 1051.

If there is a remainder the operation may be continued by annexing periods of decimal ciphers; for each period thus annexed there will be one decimal figure in the root. Thus, let it be required to find the square root of 187 to two decimal places:

OPERATION.

$\dot{1}8\dot{7}$ (13.67
1
23) 87
69
266) 180,0
159 6
2727) 2 0 40,0
1 9 08 9
71 1 . . . Remainder.

Find the square root of the following numbers to two decimal places:

7.	229.	*Ans.* 15.13.
8.	354.	*Ans.* 18.81.

The square root of a decimal may be found by the preceding rule. In this case we begin to point off periods at the decimal point and proceed toward the right.

The square root of a common fraction may be found by finding the square root of the numerator for a new numerator, and the square root of the denominator for a new denominator. If either term is not a perfect square, the fraction may be converted into a decimal and its root may then be found, by the preceding rule, to any degree of approximation.

Find the square roots of the following numbers to three places of decimals:

9.	.0249.	*Ans.* .157.
10.	.69.	*Ans.* .830.
11.	.1051.	*Ans.* .324.
12.	.0850.	*Ans.* .291.

Cube Root.

203. In accordance with the principles demonstrated in algebra, (Manual of Algebra, Art. **111**), we have the following

RULE.

I. Separate the number into periods of three figures each, beginning at the right; the left-hand period will often contain less than three figures.

II. Find the greatest perfect cube in the first period on the left, and set its root on the right after the manner of a quotient in division; subtract the cube of this root from the first period and to the remainder bring down the first figure of the next period for a dividend.

III. Take three times the square of the root thus found for a divisor, find how many times it is contained in the dividend, and place the quotient for a second figure of the root. Cube the number thus found, and, if its cube is greater than the first two periods, diminish it successively by 1 until its cube is less than the first two periods: then subtract the result from the first two periods and to the remainder bring down the first figure of the next period for a new dividend.

IV. Take three times the square of the root found for a new divisor and proceed as before, continuing the operation till the periods have been operated on.

If there is no remainder the number is a perfect cube and the root is *exact;* if there is a remainder the root is true to within 1.

EXAMPLES.

1. Find the cube root of 804357.

Explanation.—The number having been separated into periods, we find the greatest cube in 804 to be 729 and its cube root, 9, is the first figure of the root; taking 729 from 804 and bringing down 3, we have 753; dividing this by 3 times the square of 9, or 243, we get 3 for the second figure of the root; cubing 93 we find the result equal to the given number; hence 93 is the required root.

OPERATION.

$$\begin{array}{rl} & 80\dot{4}\,35\dot{7}\ (\,93 \\ 9^3 = & \underline{729} \\ 3 \times 9^2 = 243\ ) & \underline{753} \\ 93^3 = & \underline{804357} \\ & 0 \end{array}$$

The cube root of a number is indicated by the sign $\sqrt[3]{\ }$; thus, $\sqrt[3]{64} = 4$.

The sign $\sqrt{\ }$ is called the *radical sign* and the number 3 is called the *index*.

Perform the following operations:

1. $\sqrt[3]{531441}$. *Ans.* 81.
2. $\sqrt[3]{970299}$. *Ans.* 99.
3. $\sqrt[3]{35937000}$. *Ans.* 330.
4. $\sqrt[3]{224755712}$. *Ans.* 608.

If there is a remainder, the operation may be continued by annexing decimal ciphers; for each period thus annexed there will be 1 decimal place in the root.

Let the following examples be wrought to 2 decimal places:

5. $\sqrt[3]{67}$. *Ans.* 4.06.
6. $\sqrt[3]{104}$. *Ans.* 4.70.
7. $\sqrt[3]{206}$. *Ans.* 5.90.
8. $\sqrt[3]{585}$. *Ans.* 8.36.

The cube roots of common and decimal fractions may be found in a manner entirely similar to that employed in finding the square root.

EXAMINATION QUESTIONS.

(201.) What is a root? A square root? A perfect square? A cube root? A perfect cube? What is meant by extracting a root? **(202.)** Give the rule for extracting a square root. How do you extract the square root of a decimal? **(203.)** What is the rule for cube root.

III. PROGRESSIONS.

Definitions.

204. A **Progression** is a succession of numbers that increase, or decrease, according to a common law.

The numbers forming a progression are called *terms;* the first and last terms are called *extremes* and all the rest are called *means.*

Progressions are of two kinds, *arithmetical* and *geometrical.*

1°. ARITHMETICAL PROGRESSION.

Definition.

205. An **Arithmetical Progression** is one in which each term, after the first, is equal to the preceding term *increased*, or *diminished*, by a given number. This number is called the *common difference.*

If a progression is formed by the continued addition of a common difference it is *increasing*; if it is formed by the continued subtraction of a common difference it is *decreasing.*

The first of the following progressions is *increasing* and the second is *decreasing:*

2 4 6 8 10 . . . *Increasing progression.*
10 8 6 4 2 . . . *Decreasing progression.*

If an *increasing* progression is inverted, that is, if it is taken in a reverse order, it becomes a *decreasing* progression.

To Find any Term.

206. From the preceding definitions it is obvious that we may find any term by the following

RULE.

Multiply the common difference by the number of terms that precede the required term; if the progression is increasing add the product to the first term, if the progression is decreasing, subtract the product from the first term.

EXAMPLES.

1. The first term of an *increasing* arithmetical progression is 3, and the common difference is 3; what is the 9th term? *Ans.* $3 + 3 \times 8 = 27$.

2. The first term of a *decreasing* arithmetical progression is 36, and the common difference is 6; what is the 5th term? *Ans.* $36 - 6 \times 4 = 12$.

3. In an increasing progression the first term is 4, and the common difference is 2; what is the 20th term? *Ans.* 42.

4. In a decreasing progression the first term is 45, and the common difference 4; what is the 8th term? *Ans.* 17.

To Find the Sum of the Terms.

207. A rule for finding the sum of the terms may be deduced by inverting the progression and proceeding as in the following

OPERATION.

3,	6,	9,	12,	15,	18 . . .	Given progression.
18,	15,	12,	9,	6,	3 . . .	Same inverted.
21 +	21 +	21 +	21 +	21 +	21 . . .	Sum of both.

The sum of the terms in both progressions is obviously equal to twice the sum of the terms of the given progression; hence, the sum of the terms of the given progression is $\frac{21}{2} \times 6$, or 63.

Since all similar cases may be treated in the same manner, we have the following

RULE.

Multiply half the sum of the extremes by the number of terms.

EXAMPLES.

1. The first term of a progression is 3, the last term is 27, and the number of terms is 9; what is the sum of the terms?

Ans. $\frac{27+3}{2} \times 9 = 135.$

If the last term is not given, it may be found by the rule in Article **206**.

2. The first term of a decreasing progression is 36, the common difference is 6, and the number of terms is 5; what is the sum of the terms?

Ans. $\frac{36+12}{2} \times 5 = 120.$

3. In a decreasing progression, the first term is 45, the common difference is 4, and the number of terms is 8; what is the sum of the terms? *Ans.* 248.

4. What is the sum of the natural numbers, 1, 2, 3, &c., up to 99, inclusive? *Ans.* 4,950.

5. The first term of a decreasing progression is 15, the last term is 5, and the number of terms is 6; what is the sum of the terms? *Ans.* 60.

6. The first term of an increasing progression is 15, the common difference is 3, and the number of terms is 6; what is the sum of the terms? *Ans.* 135.

7. What is the sum of the terms of the progression 1, 2, 3, 4, etc., up to 12 inclusive? *Ans.* 78.

8. The first term of an increasing progression is 7, the common difference is 4, and the number of terms is 7; what is the sum of the terms? *Ans.* 133.

2°. GEOMETRICAL PROGRESSION.

Definition.

208. A Geometrical Progression is one in which each term, after the first, is equal to the preceding term *multiplied* by a given number. This number is called the *ratio* of the progression.

If the ratio is greater than 1, the progression is *increasing;* if less than 1, the progression is *decreasing.* Thus,

$$2, \quad 4, \quad 8, \quad 16,$$

is an increasing progression, and

$$16, \quad 8, \quad 4, \quad 2,$$

is a decreasing progression. The *ratio* in the first case is 2 and in the second case it is $\frac{1}{2}$; in all cases, the *ratio is equal to the quotient obtained by dividing the second term by the first.*

To Find any Term.

209. In accordance with the preceding definitions, we may find any term by the following

RULE.

Raise the ratio to a power whose exponent is the number of terms that precede the required term and multiply the first term by the result.

EXAMPLES.

1. In a progression the first term is 3, and the ratio is 3; what is the 6th term? *Ans.* $3 \times 3^5 = 729$.

2. The first term of a progression is 64, and the ratio is $\frac{1}{2}$; what is the 5th term? *Ans.* $64 \times (\frac{1}{2})^4 = 4$.

3. Find the 10th term of the progression 2, 4, 8, &c.
Ans. $2 \times 2^9 = 1{,}024$.

4. What is the 5th term of the progression 243, 81, 27, &c.?
Ans. 3.

To Find the Sum of the Terms.

210. Let it be required to find the sum of 4 terms of the series 2, 8, &c.

$2 + 8 + 32 + 128$ . . . . Indicated sum of the terms;

$\phantom{2 + {}}8 + 32 + 128 + 512$ . . . . 4 times the sum;

$512 - 2$ . . . . 3 times the sum;

$\dfrac{512 - 2}{3} = \dfrac{128 \times 4 - 2}{3} = 170$ . . . Required sum.

Explanation.—Having indicated the sum of the terms, we multiply each by 4 and set the products one place toward the right; the sum of these results is 4 times the sum of the given series; subtracting the latter from the former, we find 512 − 2, which is 3 times the required sum; dividing by 3, we have 170, which is the required sum.

Since all similar cases may be treated in like manner, we have the following

RULE.

Multiply the last term by the ratio and diminish the product by the first term; then divide the remainder by the ratio less 1.

EXAMPLES.

1. The first term of a progression is 3, the last term is 729, and the ratio is 3; what is the sum of the terms?
Ans. $\dfrac{729 \times 3 - 3}{2} = 1{,}092$.

If the first term and ratio are given, the last term may be found by the preceding rule.

2. The first term of a progression is 2, the ratio is 4, and the number of terms is 5; what is the sum of the terms? *Ans.* 682.

EXAMINATION QUESTIONS.

(204.) What is a progression? What are terms? Extremes? Means? (205.) What is an arithmetical progression? An increasing progression? A decreasing progression? (206.) What is the rule for finding any term? For finding the sum of the terms? (208.) What is a geometrical progression? What is the ratio? When is the progression increasing and when decreasing? (209.) How do you find any term? (210.) The sum of any number of terms?

IV. ANNUITIES.

Definitions.

311. An **Annuity** is a sum of money payable periodically. In what follows, we shall suppose it payable annually.

An annuity is said to be *certain* when it is payable for a definite time; it is *contingent* when it depends on some contingent event, such as the death of one or more persons.

If an annuity is payable at once, it is said to be *immediate*, or *in possession;* if it is not payable immediately, it is called a *deferred annuity*, or *an annuity in reversion.*

The **present value** of an annuity is a sum of money, which being put at compound interest will give all the sums necessary to meet the different payments.

The present value of an annuity is usually found from a table like the following:

TABLE,

Showing the present value of an annuity of $1 at various rates of interest and for any number of years up to 30.

YRS.	4%.	5%.	6%.	YRS.	4%.	5%.	6%.
1	0.962	0.952	0.943	16	11.652	10.838	10.106
2	1.866	1.859	1.834	17	12.166	11.274	10.477
3	2.775	2.723	2.673	18	12.659	11.690	10.828
4	3.630	3.546	3.465	19	13.134	12.085	11.158
5	4.452	4.329	4.212	20	13.590	12.462	11.469
6	5.242	5.076	4.917	21	14.029	12.821	11.764
7	6.002	5.786	5.582	22	14.451	13.163	12.042
8	6.733	6.463	6.210	23	14.857	13.489	12.303
9	7.435	7.108	6.802	24	15.247	13.799	12.550
10	8.111	7.722	7.360	25	15.622	14.094	12.783
11	8.760	8.306	7.887	26	15.983	14.375	13.003
12	9.385	8.863	8.384	27	16.330	14.643	13.211
13	9.986	9.394	8.853	28	16.663	14.898	13.406
14	10.563	9.899	9.295	29	16.984	15.141	13.591
15	11.118	10.380	9.712	30	17.292	15.372	13.765

1°. Certain Annuities.

212. To find the present value of an annuity in possession, we have the following

RULE.

Find from the table the present value of an annuity of $1 at the same rate and for the same time; then multiply this by the number of dollars in the annuity.

EXAMPLES.

1. What is the present value of an annuity of $125, to continue 20 years, interest being reckoned at 6%?

Solution.—Opposite 20 and under 6% we find $11.469; multiplying this by 125, we have,

$11.469 × 125 = $1,433.625. *Ans.*

2. What must be paid for an annuity of $300 for 30 years, interest being reckoned at 5%?

Ans. $15.372 × 300 = $4,611.60.

3. Find the cost of an annuity of $500 for 25 years, interest being reckoned at 4%. *Ans.* $7,811.

To find the present value of a deferred annuity we must know when the payment begins and when it ends. Knowing these, we have the following

RULE.

Find the present value up to the time the annuity ends and from the result subtract the present value up to the time it begins.

In applying the rule, the time of beginning is 1 year previous to the first payment.

EXAMPLES.

4. What is the present value of an annuity of $100, to commence after 10 years and to continue 7 years, interest being reckoned at 6%?

Solution.—The present value of $1 for 17 years at 6% is $10.477, and the present value of $1 for 10 years is $7.360; the difference between them is $3.117, and this multiplied by 100 gives

$3.117 × 100 = $311.70. *Ans.*

5. What must be paid for an annuity of $500, commencing 20 years hence, and running 10 years, interest being reckoned at the rate of 5%? *Ans.* $1,455.

2°. Contingent Annuities.

213. The most important contingency that determines the continuance of an annuity is the life of the recipient.

A **life annuity** is an annuity to continue during the life of some individual.

The following table shows the present value of a life annuity of $1 for different ages, the first payment being made at the end of 1 year:

TABLE.

AGE.	PRESENT VALUE.	AGE.	PRESENT VALUE.	AGE.	PRESENT VALUE.
31	14.324	41	13.204	51	11.585
32	14.234	42	13.074	52	11.370
33	14.137	43	12.940	53	11.148
34	14.034	44	12.803	54	10.920
35	13.929	45	12.658	55	10.681
36	13.817	46	12.506	56	10.436
37	13.702	47	12.345	57	10.184
38	13.583	48	12.175	58	9.929
39	13.458	49	11.989	59	9.679
40	13.329	50	11.795	60	9.437

This table is computed on the supposition that the rate of interest is 6%.

EXAMPLES.

1. What is the cost of a life annuity of $200 for a person aged 46?

Solution.—The present value of a life annuity of $1 for a person aged 46 is $12.506; hence, the present value of a life annuity of $200 for the same person is

$12.506 × 200 = $2,501.20. *Ans.*

2. What must be paid by a person aged 57 to purchase a life annuity of $500?

Ans. $10.184 × 500 = 5,092.

EXAMINATION QUESTIONS.

(211.) What is an annuity? An immediate annuity? A deferred annuity? What is the present value of an annuity? **(212.)** How found? **(213.)** What is a contingent annuity? A life annuity?

CHAPTER VIII.

MENSURATION.

Definition.

214. Mensuration is the operation of finding the number of units in a geometrical magnitude.

The unit of a magnitude is always a magnitude of the same kind; the unit of lines, or the *linear unit*, is a straight line of given length; as *one foot:* the unit of surfaces, or the *superficial unit*, is a square whose sides are equal to the linear unit; as *one square foot:* the *unit of volume*, or the *cubic unit*, is a cube whose edges are equal to the linear unit; as *one cubic foot*, (Arts. **128**, **131**).

The rules for mensuration depend on the definitions and principles of Geometry, some of which have been given, (Arts. **116–124**).

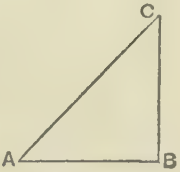
Additional Definitions.

215. A **Polygon** is a portion of a plane bounded by straight lines.

Each of the bounding lines is called a *side* of the polygon and the point at which any two sides meet is called a *vertex* of the polygon.

The *area* of a polygon is the number of superficial units it contains.

216. A **Triangle** is a polygon of three sides; as ABC; the side AB on which it is supposed to stand is called its *base*, and the perpendicular distance, CB, from the opposite vertex to the base is called its *altitude.*

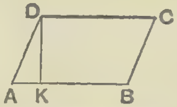


TRIANGLE.

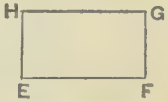
A **Right-angled Triangle** is a triangle that has one right angle. Thus, the triangle ABC is right-angled at B.

The side AC, opposite the right angle, is called the *hypothenuse.*

217. A **Parallellogram** is a polygon of four sides, parallel two and two; a **Rectangle** is a right-angled parallelogram.



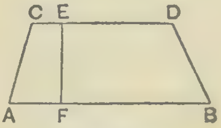
PARALLELOGRAM.



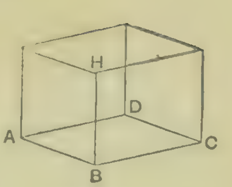
RECTANGLE.

The figure ABCD is a parallelogram whose *base* is AB and whose *altitude*, or *breadth*, is KD; the figure EFGH is a rectangle whose *base* is EF and whose *altitude*, or *breadth*, is FG.

218. A **Trapezoid** is a polygon of four sides, only two of which are parallel. The figure ABCD is a trapezoid; one of the parallel sides, AB, is its *base*, and the perpendicular distance EF is its *altitude.*



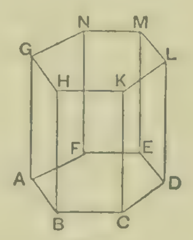
TRAPEZOID.



RECTANGULAR PARALLELOPIPEDON.

219. A **Rectangular Parallelopipedon** is a volume bounded by six rectangles, parallel two and two.

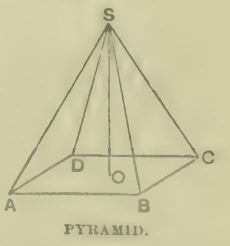
The volume ABCD–H is a rectangular parallelopipedon, whose *base* is ABCD and whose *altitude* is BH.



PRISM.

220. A **Prism** is a volume bounded by two parallel and equal polygons called *bases* and by parallelograms called *lateral faces.*

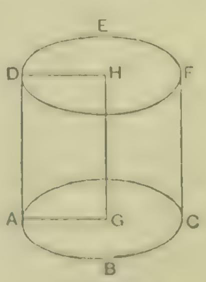
The volume ABCE–GHKM is a prism, whose *bases* are ABCDEF and GHKLMN, and whose *altitude* is the perpendicular distance between them.



PYRAMID.

221. A **Pyramid** is a volume bounded by a polygon, called the *base*, and by triangles which make up the *lateral* surface.

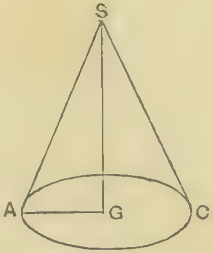
The volume S–ABCD is a pyramid whose *base* is ABCD, whose vertex is S, and whose *altitude* is the perpendicular distance SO, from the vertex to the base.



CYLINDER.

222. A **Cylinder** is a volume generated by a rectangle revolving about one of its sides. The opposite side generates the *convex surface* and the adjacent sides generate circles called *bases.*

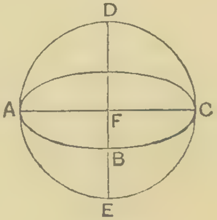
The volume ABC–DEF is a cylinder generated by the rectangle ADHG; ABC and DEF are its bases and HG is its altitude.



CONE.

223. A **Cone** is a volume generated by revolving a right-angled triangle about one of the sides adjacent to the right angle; the other side generates the *base* and the hypothenuse generates the *convex surface.*

The volume S–ABC is a cone, whose base is ABC, whose vertex is S, and whose altitude is SG.



SPHERE.

224. A **Sphere** is a volume generated by revolving a circle about one of its diameters.

The volume ABC–D is a sphere, whose generating circle is AECD; the point F is the *center* of the sphere, and the radius of the generating circle is the *radius* of the sphere.

A plane through the center of a sphere intersects it in a *great circle*, that is, in a circle whose radius is equal to the radius of the sphere.

225. The following rules are based on principles demonstrated in Geometry:

Mensuration of Lines.

1°. *To find the circumference of a circle.*

RULE.

Multiply the diameter by 3.1416.

EXAMPLES.

1. What is the length of a circumference whose diameter is 12 feet? *Ans.* 12 *ft.* × 3.1416 = 37.6991 *ft.*

2. What is the length of a circumference whose diameter is 6.75 *ft.*? *Ans.* 21.2058 *ft.*

3. Find the length of a circumference whose radius is 8.5 *in.* *Ans.* 53.4071 *in.*

2°. *To find the hypothenuse of a right-angled triangle.*

RULE.

Find the sum of the squares of the sides about the right angle and extract the square root of the result.

4. Find the hypothenuse of a right-angled triangle whose other sides are 24 feet and 18 feet.

By the term *square of a line* we mean a square whose sides are equal to the line. Thus, the square of 24 feet is a square 24 feet long and 24 feet wide; this contains 24 × 24, or 576 *square feet*; in like manner the square of 18 feet is 18 × 18, or 324 *square feet*; the sum of these is 900 *square feet*, which is the same as a square 30 feet long and 30 feet wide; hence, the required hypothenuse is 30 feet long.

In solving problems like this, we may omit the names of the units until the final result is found; thus,

Ans. $\sqrt{24^2 + 18^2} = \sqrt{900} = 30\,ft.$

5. Given the sides about the right angle equal to 13 *ft.* and 17 *ft.* to find the hypothenuse.

Ans. $\sqrt{13^2 + 17^2} = 21.4009\,ft.$

Mensuration of Surfaces.

3°. *To find the area of a triangle.*

RULE.

Multiply the base by the altitude and divide the product by 2.

EXAMPLES.

6. The base of a triangle is 8 feet and its altitude is 6 feet; what is its area? *Ans.* 48 *sq. ft.*

By the term *product of two lines* we always mean a rectangle whose length is one of the lines and whose breadth is the other. Hence we say that *the product of a line by a line is a surface.*

7. What is the area of a triangle whose base is 16 yards and whose altitude is 3½ yards? *Ans.* 28 *sq. yds.*

4°. *To find the area of a parallelogram.*

RULE.

Multiply the base by the altitude.

EXAMPLES.

8. The base of a parallelogram is 14 yards and its altitude is 5 yards; what is its area?

Ans. 14 *yds.* × 5 *yds.* = 70 *sq. yds.*

9. Find the area of a parallelogram whose base is 13 feet and whose altitude is 7½ feet. *Ans.* 97½ *sq. ft.*

5°. *To find the area of a trapezoid.*

RULE.

Multiply half the sum of its parallel sides by its altitude.

EXAMPLES.

10. The parallel sides of a trapezoid are 14 *yds.* and 20 *yds.* and the altitude is 7 *yds.*; what is its area?

Ans. $\frac{14\ yds. + 20\ yds.}{2} \times 7\ yds. = 119\ sq.\ yds.$

11. Find the area of a trapezoid whose parallel sides are 18 *ft.* and 22 *ft.* and whose altitude is 17 *ft.*

Ans. 340 *sq. ft.*

6°. *To find the area of a circle.*

RULE.

Multiply the square of its radius by 3.1416.

EXAMPLES.

12. What is the area of a circle whose radius is 13 inches?

Ans. 13 *in.* × 13 *in.* × 3.1416 = 530.9292 *sq. in.*

13. Find the area of a circle whose radius is 2.5 rods.

Ans. 19.635 *sq. rods.*

7°. *To find the surface of a sphere.*

RULE.

Multiply 4 times the square of its radius by 3.1416.

EXAMPLES.

14. What is the area of the surface of a sphere whose radius is 12 inches?

Ans. 4 × 12 *in.* × 12 *in.* × 3.1416 = 1809.5574 *sq. in.*

15. Find the area of the surface of a sphere whose radius is 4 feet. *Ans.* 201.0619 *sq. ft.*

8°. *To find the volume, or content, of a parallelopipedon, prism, or cylinder.*

RULE.

Multiply the base by the altitude.

EXAMPLES.

16. The base of a parallelopipedon is 24 *sq. ft.* and its altitude is 8 feet; what is its content?

Ans. 192 *cu. ft.*

By the term *product of a surface and line* we always mean a parallelopipedon whose base is equal to the surface and whose altitude is equal to the line. The number of cubic units in the volume is equal to the number of superficial units in the surface multiplied by the number of linear units in the height. Hence, we say, *the product of a surface and line*, or *the continual product of three lines, is a volume.*

17. The base of a parallelopipedon is 81 square feet and its altitude is 4 feet; what is its volume?

Ans. 324 *cu. ft.*

18. Find the content of a prism whose base is 86 *sq. ft.* and whose altitude is 7 *ft.* *Ans.* 602 *cu. ft.*

19. The base of a cylinder is equal to 80 *sq. ft.* and its altitude is equal to 5 *ft.*; what is its content?

Ans. 400 *cu. ft.*

The area of the base may be found by previous rules.

9°. *To find the content of a pyramid, or cone.*

RULE.

Multiply the base by the altitude and divide the product by 3.

EXAMPLES.

20. The base of a pyramid is 49 *sq. ft.* and its altitude is 4 *ft.*; what is its volume?

$$Ans.\ \frac{49\,sq.\,ft. \times 4\,ft.}{3} = 65.3333\,cu.\,ft.$$

21. The base of a cone is 15.9 *sq. ft.* and its altitude is 6 *ft.*; what is its content? *Ans.* 31.8 *cu. ft.*

10°. *To find the volume of a sphere.*

RULE.

Multiply 4 times the cube of the radius by 3.1416 and divide the product by 3.

EXAMPLES.

22. The radius of a sphere is 5 *ft.*; what is its volume?

$$Ans.\ \frac{4 \times 5\,ft. \times 5\,ft. \times 5\,ft. \times 3.1416}{3} = 523.6\,cu.\,ft.$$

23. Find the volume of a sphere whose radius is 11.5 *ft.* *Ans.* 6370.6263 *cu. ft.*

Board Measure.

226. A board foot is a volume one foot long, one foot wide, and one inch thick. It is equal to one twelfth of a cubic foot.

This unit is used in measuring *boards, planks*, and some kinds of timber.

Boards and planks are of uniform thickness throughout, but they are often of different widths at the two ends; in this case the half sum of the widths at the ends is taken as the width of the board, or plank. The width and thickness are usually expressed in inches, but the length is given in feet.

To find the number of board feet in a board, plank, or stick of timber of uniform thickness.

RULE.

Multiply the length in feet by the product of the breadth and thickness, both in inches, and divide the result by 12.

EXAMPLES.

1. How many board feet in a board 13 feet long, 16 inches wide, and $1\frac{1}{4}$ inches thick?

Ans. $\dfrac{13 \times 16 \times 1\frac{1}{4}}{12} = 21\frac{2}{3}$.

2. A board is 17 feet long, 13 inches wide at one end, 17 inches wide at the other end, and 1 inch thick; how many board feet does it contain? *Ans.* $21\frac{1}{4}$.

3. A plank 16 feet long and $2\frac{1}{2}$ inches thick is 16 inches wide at one end and 18 inches wide at the other; how many board feet does it contain? *Ans.* $56\frac{2}{3}$.

4. How many board feet in a piece of scantling 18 feet long, 4 inches thick and 9 inches wide?

Ans. 54.

Timber Measure.

227. Timber, when not expressed in board measure, is usually given in cubic feet.

Timber may be *round*, that is, it may have a *circular cross section;* or it may be *hewn*, that is, it may have a *rectangular cross section.*

The cross section may be the same throughout, or it may be greater at one end than at the other. The *mean cross section* is the cross section midway between the two ends.

The cross section of a round stick of timber at any point can be found when we know its *girt* at that point. The *girt* is the circumference after the bark is removed.

The cross section in square inches may be found at any point where the girt is known by the following

RULE.

Multiply the square of the girt in inches by .0796.

EXAMPLES.

1. The girt of a round stick of timber is 42 inches; what is its cross section?

Ans. $42^2 \times .0796 = 140.4144$ *sq. in.*

2. Find the cross section of a round stick at a point where the girt is 52 inches. *Ans.* 215.2384.

3. If the girt is 60 inches, what is the cross section?

Ans. 286.56 *sq. in.*

The cross section of a hewn stick of timber at any point may be found by the following

RULE.

Multiply the breadth of the stick by its thickness at that point, both in inches; the product is the cross section in square inches.

EXAMPLES.

4. The breadth of a square stick of timber at its larger end is 14 inches and its thickness is 13 inches; what is its greatest cross section?

Ans. 14 *in.* × 13 *in.* = 182 *sq. in.*

5. The breadth of the same stick at its smaller end is 12 inches and its thickness is 10 inches; what is its smallest cross section? *Ans.* 120 *sq. in.*

6. The breadth of the same stick at the middle of its length is 13 inches and its thickness is $11\frac{3}{4}$ inches; what is its mean cross section? *Ans.* $152\frac{3}{4}$.

Knowing the two end sections and the mean section in square inches, and the length in feet, we can find the number of cubic feet in a stick of timber by the following

RULE.

To the sum of the end sections add four times the mean section, all in square inches and multiply the result by the length in feet; then divide by 864, and the quotient will be the number of cubic feet required.

EXAMPLES.

7. The end sections of a stick of timber are 182 and 120 *sq. in.*, the middle section is $152\frac{3}{4}$ *sq. in.*, and its length is 30 *ft.*; what is its content?

$$\textit{Ans.}\ \frac{(182 + 120 + 4 \times 152\frac{3}{4}) \times 30}{864} = 31.7014\,cu.\,ft.$$

8. The end sections of a stick of timber 40 *ft.* long are 460 and 400 *sq. in.*, and the mean section is 440 *sq. in.*; what is its content? *Ans.* 121.2963 *cu. ft.*

Wood Measure.

228. A **cord foot** is a volume 4 feet wide, 4 feet high, and 1 foot long. It is equal to 16 cubic feet.

A **cord** is equal to 8 cord feet, or to 128 cubic feet.

Wood for fuel is usually cut into lengths of 4 feet and laid up in rectangular piles 4 feet in height. To find the number of cords in such a pile we have the following

RULE.

Divide the length of the pile in feet by 8.

EXAMPLES.

1. A pile of wood is 4 feet thick, 4 feet high, and 18 feet long; how many cords does it contain?

Ans. $\frac{18}{8} = 2\frac{1}{4}$.

2. A similar pile is 27 feet long; how many cords does it contain? *Ans.* $3\frac{3}{8}$.

To find the number of cords in any rectangular pile of wood, we have the following

RULE.

Find the continued product of the length, breadth, and height, expressed in feet, and divide the result by 128.

EXAMPLES

3. A pile of wood is 30 feet long, $3\frac{1}{2}$ feet broad and 5 feet high; how many cords does it contain?

Ans. $\frac{30 \times 3\frac{1}{2} \times 5}{128} = 4\frac{13}{128}$.

In expressing the dimensions of wood for fuel it will be sufficiently accurate to give them to the nearest tenth of a foot.

4. A pile of wood is $14\frac{1}{2}$ feet long, $5\frac{1}{2}$ feet wide, and $6\frac{1}{2}$ feet high; how many cords does it contain?

Ans. $\frac{14\frac{1}{2} \times 5\frac{1}{2} \times 6\frac{1}{2}}{128} = 4\frac{51}{1024}$.

5. A pile of wood is 37 *ft.* 4 *in.* long, 3 *ft.* 11 *in.* thick, and 5 *ft.* 6 *in.* high; how many cords does it contain?

$$Ans.\ \frac{37.3 \times 3.9 \times 5.5}{128} = 6.25\ cords.$$

Method of Duodecimals.

229. If the linear dimensions are expressed in feet and inches, areas and volumes may be found by the method of duodecimals.

In this system of numbers the primary unit is 1 *foot*; it may be a *linear foot*, a *square foot*, or a *cubic foot*.

One twelfth of a foot is called a *prime*, one twelfth of a prime is called a *second*, and one twelfth of a second is called a *third*, as shown in the following

TABLE.

12 thirds ‴	make	1 second . . .	″.
12 seconds	"	1 prime. . . .	′.
12 primes	"	1 foot.	*ft.* *sq.ft.* *cu.ft.*

The scale of the system both ascending and descending is

12, 12, 12.

In accordance with the principles laid down in Art. **125** for multiplying lines by lines, and surfaces by lines, we see that

feet	multiplied by	*feet*	give	*feet*;
feet	" "	*primes*	"	*primes*;
feet	" "	*seconds*	"	*seconds*;
primes	" "	*primes*	"	*seconds*;
primes	" "	*seconds*	"	*thirds*.

Operation of Multiplication.

230. Let it be required to find the continued product of 3 *ft.* 5 *in.*, 2 *ft.* 6 *in.*, and 4 *ft.* 7 *in.*:

OPERATION.

ft.	′	″	‴
3	5		
2	6		
6	10		
	18	30	
8	6	6	
4	7		
32	24	24	
	56	42	42
39	1	9	6

$= 39\frac{43}{288}$ *cu. ft.*

Explanation.—Having written the first two numbers so that units of the same name stand in the same column, we begin at the left hand and multiply all the parts of the multiplicand by 2, writing the products, without reduction, in their proper columns according to the principles explained in the last article. We then multiply all the parts of the multiplicand by 6, and place the products in their proper columns, as determined by the rules in the last article. We next add the partial products by the rule for compound addition, which gives 8 *sq. ft.* 6′ 6″.

We now multiply 8 *sq. ft.* 6′ 6″ by 4 *ft.* 7 *in.*, in the same manner as before and find for the required product 39 *cu. ft.* 1′ 9″ 6‴, that is, $(39 + \frac{1}{12} + \frac{9}{144} + \frac{6}{1728})$ *cu. ft.*, which is equal to $39\frac{43}{288}$ *cu. ft.*

In like manner we may multiply in all similar cases; hence, the following

RULE.

I. Write the numbers so that units of the same name shall stand in the same column.

II. Multiply all the parts of the multiplicand by each part of the multiplier and write the corresponding partial products, without reduction, in their proper columns.

III. Add the partial products by the rule for compound addition.

EXAMPLES.

	(1.) ft.	(1.) ′	(1.) ″	(2.) ft.	(2.) ′	(2.) ″
Multiplicand . .	3	2		5	7	
Multiplier	5	7		7	10	
	15	10		35	49	
		21	14		50	70
Product	17	8	2	43	8	10
	$= 17\frac{49}{72}$ *sq. ft.*			$= 43\frac{53}{72}$ *sq. ft.*		

3. Multiply 3 *ft.* 7 *in.* by 9 *ft.* 4 *in.*

Ans. 33 *sq. ft.* 5′ 4″ $= 33\frac{4}{9}$ *sq. ft.*

4. Multiply 5 *ft.* 11 *in.* by 16 *ft.* 2 *in.*

Ans. 95 *sq. ft.* 7′ 10″ $= 95\frac{47}{72}$ *sq. ft.*

5. Find the continued product of 3 *ft.* 4 *in.*, 2 *ft.* 7 *in.*, and 6 *ft.* 11 *in.* *Ans.* 59 *cu. ft.* 6′ 8″ 8‴ $= 59\frac{121}{216}$ *cu. ft.*

6. Find the continued product of 4 *ft.* 3 *in.*, 5 *ft.* 2 *in.*, and 6 *ft.* 5 *in.* *Ans.* 140 *cu. ft.* 10′ 9″ 6‴ $= 140\frac{259}{288}$ *cu. ft.*

PRACTICAL PROBLEMS.

1. How many square feet in a ceiling 17 *ft.* 3 *in.* long and 11 *ft.* 5 *in.* wide? *Ans.* $196\frac{31}{36}$ *sq. ft.*

2. How many square feet in a pavement 12 *ft.* 6 *in.* long and 10 *ft.* 2 *in.* wide? *Ans.* $127\frac{1}{12}$ *sq. ft.*

3. Find the capacity of a box 3 *ft.* 3 *in.* long, 2 *ft.* 9 *in.* wide, and 1 *ft.* 11 *in.* deep. *Ans.* $17\frac{25}{192}$ *cu. ft.*

4. Find the contents of a stick of timber 42 *ft.* 6 *in.* long, 1 *ft.* 7 *in.* wide, and 1 *ft.* 4 *in.* thick.

Ans. $89\frac{13}{18}$ *cu. ft.*

5. What is the capacity of a bin 7 *ft.* 3 *in.* long, 4 *ft.* 2 *in.* wide, and 3 *ft.* 5 *in.* deep? *Ans.* $103\frac{61}{288}$ *cu. ft.*

6. How many cords of wood in a pile 13 *ft.* 3 *in.* long, 4 *ft.* 2 *in.* wide, and 3 *ft.* 6 *in.* high?

Ans. 1 *C.* 65$\frac{11}{48}$ *cu. ft.*

7. How many cords in a pile 20 *ft.* 4 *in.* long, 4 *ft.* 3 *in.* wide, and 5 *ft.* 2 *in.* high?

Ans. 3 *C.* 62$\frac{35}{72}$ *cu. ft.*

EXAMINATION QUESTIONS.

(214.) What is mensuration? **(215.)** What is a polygon? Sides? Vertex? Area? **(216.)** What is a triangle? Define the base. The altitude. What is a right-angled triangle? Describe its parts. **(217.)** Define a parallelogram. A rectangle. Its altitude. **(218.)** What is a trapezoid? Its bases and altitude? **(219.)** What is a rectangular parallelopipedon? **(220.)** What is a prism? Its bases? Its altitude? **(221.)** What is a pyramid? Its base? Its altitude? **(222.)** What is a cylinder? Describe its parts. **(223.)** What is a cone? Describe its parts. **(224.)** What is a sphere? Describe its parts. **(225.)** Give the rule for finding the circumference of a circle. The hypothenuse of a right-angled triangle. The area of a triangle. The area of a parallelogram. The area of a trapezoid. The area of a circle. The surface of a sphere The contents of a parallelopipedon, prism, or cylinder. The contents of a pyramid, or cone. The volume of a sphere. **(226.)** What is a board foot? How do you find the number of feet in a board or plank? **(227.)** What is the method of measuring timber? Round and square? What is the girt? Rule for finding the girt? Rule for finding cross section of a round, or square stick? **(228.)** What is a cord foot? A cord of wood? **(229.)** Explain the system of duodecimals. Give the rule for multiplying lines by lines. **(230.)** What is the rule for multiplying duodecimals?

MISCELLANEOUS EXAMPLES.

1. Find the product of the sum and difference of 25 and 16. *Ans.* 369.

2. Divide the difference between 1296 and 441 by the sum of 36 and 21. *Ans.* 15.

3. Resolve 3,990 into prime factors. *Ans.* 2 . 3 . 5 . 7 . 19.

4. What are the prime factors of 9,800? *Ans.* 2 . 2 . 2 . 5 . 5 . 7 . 7.

5. Find the *g. c. d.* of 2,290 and 458. *Ans.* 458.

6. What is the *g. c. d.* of 1,435, 1,085, and 2,135? *Ans.* 35.

7. Find the *l. c. m.* of 15, 18, 24, 40, and 50. *Ans.* 1,800.

8. What is the *l. c. m.* of 508 and 889? *Ans.* 3,556.

9. Add $\frac{2}{5}$, $\frac{7}{8}$ of $\frac{3}{7}$, and $\frac{34}{25}$. *Ans.* $2\frac{27}{200}$.

10. Subtract $\frac{1}{3}$ of $4\frac{1}{2}$ from $\frac{2}{7}$ of $9\frac{1}{4}$. *Ans.* $1\frac{1}{7}$.

11. Multiply $\frac{1}{5}$ of $2\frac{1}{3}$ by $\frac{1}{8}$ of $3\frac{1}{7}$. *Ans.* $\frac{11}{60}$.

12. Divide $2\frac{1}{2}$ by $1\frac{4}{5}$. *Ans.* $1\frac{7}{18}$.

13. Multiply the sum of 3.31 and 4.06 by the difference of 8.13 and 3.43. *Ans.* 34.639.

14. Divide the sum of 3.8 and 2.05 by the difference of 8.6 and $3\frac{3}{4}$. *Ans.* 1.2268.

15. A man bought a horse and carriage; the horse cost $\frac{3}{5}$ as much as the carriage, and both together cost \$640; what was the cost of the horse? *Ans.* \$240.

16. At a certain election the successful candidate had a majority of 120, which was $\frac{1}{9}$ of all the votes cast; how many votes did the opposing candidate receive? *Ans.* 480.

17. Divide \$357 among A., B., and C., so that B. shall receive $2\frac{1}{2}$ times as much as A., and C. as much as A. and B. together.

Ans. A. has \$51, B. has \$127.50, and C. has \$178.50.

18. A. can do a piece of work in 3 days, B. can do it in 4 days, and C. can do it in 5 days; how long will it take them to do it together? *Ans.* $1\frac{13}{47}$ *da.*

19. How many bushels of oats can be raised on $4\frac{1}{2}$ acres, if each acre produces 47 *bu.* 3 *pks.*?

Ans. 214 *bu.* 3 *pks.* 4 *qts.*

20. How many bushels of wheat at \$1.75 per bushel will it take to pay for 3 *cwt.* of pork at \$7 per *cwt.*?

Ans. 12.

21. A grocer mixes 120 *lbs.* of sugar at 10 *cts.* a pound, 140 *lbs.* at 12 *cts.*, and 60 *lbs.* at 14 *cts.*; at what rate must he sell it to clear 20% on its cost?

Ans. $13\frac{19}{20}$ *cts.*

22. Divide \$1,000 among 3 persons in the proportion of 5, 7, and 8. *Ans.* \$250, \$350, and \$400.

23. Bought a horse for \$312 and sold him at a loss of $12\frac{1}{2}$%; what did I receive? *Ans.* \$273.

24. A man travels 100 miles by rail and 100 miles by stage; his average rate of travel is 16 miles per hour and the rate of the train is 40 miles per hour; what is the rate of the stage? *Ans.* 10 *miles per hour.*

25. A tank 7 *ft.* deep, 12 *ft.* long, and 9 *ft.* broad is full of water; what is the weight of the water, if each cubic foot weighs $62\frac{1}{2}$ *lbs.*? *Ans.* 47,250 *lbs.*

26. Three men can do a piece of work in 6 days; the first alone can do it in 15 days, and the second alone

can do it in 12 days; how long would it take the third alone to do it? *Ans.* 60 days.

27. A man bequeathed $37,000 to his family; he gave $\frac{1}{4}$ to his wife, $\frac{1}{5}$ to his son, and divided the rest equally between 5 daughters; how much did each daughter receive? *Ans.* $4,070.

28. A merchant purchased cloth to the amount of $1,250 and silk goods to the amount of $900; he sold the former at a profit of 20% and the latter at a loss of 10%; how much did he gain by the operation?

Ans. $160.

29. A. B., and C. enter into partnership; A. puts in $\frac{1}{3}$ of the capital, B. puts in $\frac{1}{4}$ of the capital, and C. puts in the rest; at the end of the year their profit amounts to $10,440; what is C.'s share of the profit?

Ans. $4,350.

30. I bought a lot for $700 and after holding it for 1 year, sold it at an advance of 20% on the cost and interest at 7%; what did I get? *Ans.* $898.80.

31. A merchant bought cloth to the amount of $1,500 and sold it again for $1,770; what was his gain %?

Ans. 18%.

32. A merchant bought goods for $2,000 cash, and sold them again for $2,540 on a credit of four months; what was his gain % in addition to the interest of his money? *Ans.* 25%.

33. A merchant sells cloth at 3.12\frac{1}{2}$ per yard and clears 25%; how much would he clear if he were to sell it for $3.50 per yard? *Ans.* 40%.

34. Two couriers start together from the same point and travel in the same direction; the first travels 23

miles in 2½ hours and the second travels 11 miles in 2¼ hours; how far apart are they at the end of 31½ hours?
Ans. 53 *miles.*

35. A laborer spent 25% of his week's wages for flour and had $11.25 left; what did he receive per week?
Ans. $15.

36. John Churchill's farm is composed of 36 acres of pasture land, 22 acres of meadow land, 18 acres of plough land and 20 acres of woodland; supposing it to be rectangular in shape and 128 rods long, what is its width? *Ans.* 120 rods.

37. If 30 bushels of wheat cost $67.50, how much can be bought for $438.75? *Ans.* 195 bushels.

38. If 12 men can build a wall in 20 days, how many men will be required to do the same work in 8 days?
Ans. 30 *men.*

39. If $100 gain $6 in 9 months, what principal will gain $11 in 5 months? *Ans.* $330.

40. A certain quantity of hay will feed 963 sheep for 7 weeks; how many must be turned away that it may feed the remainder for 9 weeks? *Ans.* 214.

41. The third part of an army were killed, the fourth part were taken prisoners, and there remained 10,800; how many men did the army contain?
Ans. 25,920 *men.*

42. Thomas sold 600 pineapples at 16⅔ *cts.* each, and received as much for them as Henry did for a certain number of melons which he sold at 40 *cts.* apiece; how many melons did Henry sell? *Ans.* 250.

43. A flag-staff stands ⅙ of its length in the ground,

12 feet in the water, and $\frac{5}{6}$ of its length in the air; what is the length of the staff? *Ans.* 216 *ft.*

44. What is the difference between the interest on $584 for 66 days at 6% and the interest on $500 for 63 days at 7%? *Ans.* $0.299.

45. A shipper insures $2,500 worth of oats, paying 4% for insurance; what must be the face of his insurance that he may receive the value of his oats and the cost of insurance in case of loss?

Ans. $\frac{\$2,500}{.96} = \$2,604\frac{1}{6}$.

46. A merchant insures $4,000 worth of silk at $2\frac{1}{2}$%; what must be the face of his insurance that he may lose nothing in case of its destruction by fire?

Ans. $\$4,102\frac{22}{39}$.

47. A merchant bought several bales of cloth, each containing $133\frac{1}{3}$ *yds.*, at the rate of 12 *yds.* for $11, and sold it at the rate of 8 *yds.* for $7, losing $100 by the transaction; how many bales did he buy? *Ans.* 18.

48. A. owes B. $2,500 payable in 4 months, but at the end of 3 months he pays him $1,500; how long after this payment before the balance is equitably due?

Ans. $2\frac{1}{2}$ *months.*

49. What is the bank discount on an accommodation note of $3,000 for 60 days at 7%? *Ans.* $36.75.

50. A lady wishes to carpet a floor 15 *ft.* 9 *in.* wide and 22 *ft.* 6 *in.* long, with carpeting $\frac{3}{4}$ *yd.* wide; if the carpeting is worth $2.50 per *yd.* how much will it cost?

Ans. $131.25.

51. A box is 4 *ft.* 9 *in.* long, 3 *ft.* 6 *in.* wide, and 2 *ft.* 9 *in.*

deep; how many bushels will it contain if a bushel is equal to 2,150⅖ cubic inches? *Ans.* 33.5157 *bu.*

52. How many board feet in 250 planks, each 14 *ft.* long, 16 *in.* wide and 2½ *in.* thick?

Ans. 11,666⅔ *board ft.*

53. A pile of wood is 3½ feet wide, 5¼ feet high, and and 147 feet long; how many cords does it contain?

Ans. 21 *C.* 13⅛ *cu. ft.*

54. A., B., and C. undertake a certain job for $400: A. furnishes 4 men for 8 days, B. furnishes 6 men for 7 days, and C. furnishes 13 men who finish the job in 2 days; what share of the money ought each to receive?

Ans. A. should receive $128.
B. " " $168.
C. " " $104.

55. A merchant bought a piece of merino containing 32 yards for $25.60, and then marked it so that he could fall 4% on the asking price and still make 20% on its cost; what must he mark it per yard?

Ans. $1 *per yd.*

56. A house is 40 feet from the ground to the eaves, and a ladder is placed with its foot 30 feet from the house; how long must it be to reach the eaves?

Ans. 50 *ft.*

57. A man buys ⅝ of a piece of property and sells 20% of his share for $5,000, clearing 25% on its cost; what was the original value of the whole property?

Ans. $32,000.

58. If wheat at 8*s.* 3*d.* per bushel gives a profit of 10%, how much will it give if sold at 9*s.* 4½*d.* per bushel?

Ans. 25%.

59. Of the trees in a certain orchard $\frac{1}{2}$ are apple trees, $\frac{1}{4}$ peach trees, $\frac{1}{6}$ plum trees, and the remaining 200 are cherry trees; how many trees are there in the orchard? *Ans.* 2,400.

60. If a quantity of bread will last 1,500 men for 12 weeks at the rate of 20 *oz.* per day for each man, how long will the same bread last 2,500 men at the rate of 16 *oz.* per day for each man? *Ans.* 9 *weeks.*

61. A path 3 feet wide runs around a rectangular yard whose length is 105 *yds.* and whose breadth is 95 *yds.*; if the outer edge of the walk is everywhere 4 feet from the wall of the yard, how many square feet will it contain? *Ans.* 3,468 *sq. ft.*

62. A man leaves $38,000 to be divided among 3 sons and 3 daughters; each son is to receive 33⅓% more than the eldest daughter and each of the younger daughters is to receive 33⅓% less than the eldest; what is the share of each?

Ans.
Each son . . . . $8,000.
Eldest daughter . . . . . . $6,000.
Each younger daughter . $4,000.

63. If a clock beats 31 times in 30 seconds, how many times will it beat in 3 *da.* 5 *hrs.* 4 *min.*? *Ans.* 286,688.

64. A man agrees to execute a certain contract in 60 days and places 30 men on the work; at the end of 48 days the job is but half completed; how many men must he employ for the rest of the time to fulfill his contract? *Ans.* 120 *men.*

65. A man sells eggs; to the first person he sells half his stock and *one* more, to the second person he

sells half of what remains and *one* more, and to the third person he sells half of what remains and *one* more, when he has none left; how many had he at first?

Ans. 14.

66. At what time between 5 and 6 o'clock are the hour and minute hands of a clock together?

Ans. 5 *hrs.* $27\frac{3}{11}$ *min.*

67. If 30 men require 40 days to do a piece of work, how many men will be required to do 5 times as much work in one fifth of the time? *Ans.* 750 *men.*

68. A gentleman being asked his age, replied, if you add to it its half, its third, and three times three, the sum will be 130; what was his age? *Ans.* 66.

69. A. and B. together can do a piece of work in 18 days, but with the assistance of C. they can do it in 11 days; in what time could C. do it by himself?

Ans. $28\frac{2}{7}$ *days.*

70. A. starts from Bantam at 9 *h.* 15 *m.* A. M. and travels toward Norfolk at the rate of 4 miles per hour; B. starts from Norfolk at 9 *h.* 30 *m.* A. M. and travels toward Bantam at the rate of $3\frac{1}{2}$ miles per hour; the distance between the two places is 21 miles: at what time will they meet? *Ans.* 12 *h.* 10 *m.* P. M.

ANSWERS TO EXAMPLES.

The references are to both the article and the page at which the example is found.

The number in the first column is the number of the example, and that in the second column is the corresponding answer.

SIMPLE ADDITION.

ART. **20**,
pp. 21–24.

9. 14,776.
10. 11,803.
11. 52,026.
12. 11,594 *ft.*
13. 151,275.
14. 7,618 *yds.*
15. 16,082 *da.*
16. 1,038,957.
21. 82,391.
22. 779,264 *yds.*
23. $1,624,249.
24. $686,853.
25. 273,329.
26. 253,693 *ft.*
27. $1,041,262.
28. 564,407.
29. $351,405.
30. 206,317.
31. 13,507.
32. $839.
33. 171,593 *yds.*
34. 1,715,099.
35. 1,715,369.
36. 14,759,180.
37. 2,159,170.
38. 1,707,521.
39. 6,982,126.
40. 12,433,713.
41. 7,921,317.
42. 94,370,040.
43. $954.32.
44. $1,577.17.
45. $835.44.
46. $2,491.89.

ART. **21**,
pp. 24–26.

9. $16,985.
10. 107,683 *bu.*
11. 587 *mi.*
12. 258,928 *ft.*
13. 3,695 *pages.*
14. $92,950.
15. 3,073,134.

SIMPLE SUBTRACTION.

ART. **26**,
pp. 29–33.

17. 19,238.
18. 37,486.
19. 111,530.
20. 591,203.
21. 666,667.
22. 78,004.
23. 999,497.
24. 305,106.
25. 409,095.
26. 561,906,000.
27. 604,918.
28. 19,712.
29. 402,760.
30. 39,990,990.
31. 61,303.

32. 78,325.
33. 5,322 *yds.*
34. 28,571 *ft.*
35. $722,996.
36. $206,992.
37. $801,965.
38. 33,522 *yds.*
39. $57,838,447.
40. $4,312,956.
41. 2,911,106.
42. 178,514.
43. 39,499.
44. 4,880,874.
45. 1,815,309.
46. 8,600,090.
47. 959,820 *ft.*
48. 481,605.
49. 520,619.
50. 10,875.
51. 17,138.
52. 33,335.
53. $147,665.
54. 74,825 *ft.*
55. 345,153.

Practical Problems.

5. 3,602.
6. 71,837.
7. 30,388.
8. 38,788.
9. $52,806.
10. $36,861.
11. $397.
12. $883.
13. $2,004.
14. $577.
15. 9,417 *A.*
16. $25,600.
17. $9,112.
18. $6,740.
19. 1897.
20. 28,887 *bu.*
21. He lost $2,410.

SIMPLE MULTIPLICATION.

ART. 32,
pp. 39–40.

23. 305,375.
24. $398,088.
25. 131,794.
26. 776,223.
27. 536,724.
28. 337,770.
29. 207,648.
30. 598,885.
35. 6,786,000.
36. 27,318,000.
37. 33,948,000.
38. 64,090,000.
39. 16,412,400.
40. 9,396,000.
41. 6,977,400.
42. 304,741,000.
43. 326,821,000.
44. 87,464.
45. 306,432.
46. 796,176.
47. 2,399,544.
48. 2,500,627.
49. 7,833,192.
50. 68,451,100.
51. 10,367,716.
52. 20,251,891.
53. 7,812,790 *yds.*
54. 160,018,245.
55. 14,925,152.
56. $31,482,164.
57. 25,501,584.
58. 6,527,124.
59. $17,258,190.

ART. 34.
pp. 41–43.

2. 74,508.
3. 242,235.
4. 3,672,672 *yds.*
5. 1,034,352 *lbs.*
6. 35,328.
7. 12,155.
8. 229,880.

Practical Problems.

6. 119,568 *men.*
7. 16,000 *rds.*
8. $16,362,500.
9. 1,624 *bu.*
10. 4,111,200 *ft.*
11. 17,920 *rds.*
12. 51,574 *trees.*
13. 153,180.
14. 349,860 *lbs.*
15. 95,040 *ft.*
16. 40,824 *peaches*
17. 112,000.
18. 375,000.

SIMPLE DIVISION.

ART. 40,
p. 48.

25. 119$\frac{1}{4}$.
26. 177$\frac{1}{8}$.
27. 1,041.
28. 1,023.
29. 2,085.
30. 1,769.
31. 74,074.

ADDITION OF FRACTIONS.

ART. **75**, *p. 87.*

6. $1\frac{1}{3}$.

7. $2\frac{25}{252}$.

8. $1\frac{17}{60}$.

9. $15\frac{33}{35}$.

10. $10\frac{1}{60}$.

11. $2\frac{257}{360}$.

12. $2\frac{17}{42}$.

13. $2\frac{221}{270}$.

14. $2\frac{5}{16}$.

15. $2\frac{197}{840}$.

19. $22\frac{1}{11}$.

20. $672\frac{7}{33}$.

21. $624\frac{7}{8}$.

22. $106\frac{2}{57}$.

23. $1357\frac{37}{100}$.

24. $134\frac{7}{48}$.

25. $11\frac{119}{120}$.

26. $20\frac{89}{180}$.

27. $15\frac{1}{18}$.

28. $13\frac{13}{24}$.

29. $8\frac{7}{135}$.

30. $14\frac{23}{60}$.

31. $23\frac{39}{58}$.

32. $26\frac{5}{14}$.

33. $3\frac{11}{28}$.

34. $24\frac{11}{18}$.

35. $26\frac{23}{36}$.

36. $9\frac{23}{102}$.

37. $10\frac{13}{220}$.

38. $9\frac{9}{140}$.

39. $19\frac{62}{175}$.

40. $10\frac{31}{195}$.

SUBTRACTION OF FRACTIONS.

ART. **77**, *p. 89.*

5. $\frac{3}{8}$.

6. $\frac{17}{99}$.

7. $\frac{1}{36}$.

8. $\frac{1}{12}$.

9. $4\frac{7}{120}$.

10. $1\frac{7}{8}$.

11. $4\frac{1}{12}$.

12. $3\frac{1}{48}$.

13. $11\frac{7}{12}$.

14. $65\frac{7}{22}$.

15. $31\frac{11}{24}$.

16. $\frac{2}{15}$.

17. $52\frac{3}{4}$.

18. $84\frac{19}{20}$.

19. $\frac{41}{120}$.

20. $\frac{1}{2}$.

21. $1\frac{13}{24}$.

22. $4\frac{23}{54}$.

23. $\frac{31}{240}$.

24. $\frac{13}{56}$.

25. $8\frac{1}{18}$.

26. $2\frac{11}{45}$.

27. $\frac{14}{175}$.

28. $3\frac{7}{12}$.

29. $6\frac{9}{11}$.

30. $5\frac{7}{12}$.

31. $749\frac{19}{62}$.

32. $\frac{461}{2310}$.

MULTIPLICATION OF FRACTIONS.

ART. **79**, *pp. 92–93.*

12. $\frac{16}{45}$.

13. $\frac{56}{255}$.

14. $\frac{12}{55}$.

15. $\frac{7}{60}$.

16. $7\frac{13}{21}$.

17. $9\frac{23}{27}$.

18. $65\frac{5}{8}$.

19. $451\frac{11}{15}$.

20. $2\frac{4}{5}$.

21. 66.

22. $70\frac{5}{12}$.

23. $\frac{19}{66}$.

24. $121\frac{1}{2}$.

25. $\frac{17}{40}$.

28. $\frac{3}{5}$.

29. $2\frac{3}{11}$.

30. $21\frac{3}{32}$.

31. $13\frac{5}{9}$.

32. 6.

33. $4\frac{44}{355}$.

34. $9{,}333\frac{359}{430}$.

35. $\frac{255}{1276}$.

36. $1\frac{1}{4}$.

37. $1\frac{19}{51}$.

38. $1\frac{557}{675}$.

39. $24{,}450\frac{2}{7}$.

40. $\frac{2}{17}$.

41. $561\frac{33}{64}$.

DIVISION OF FRACTIONS.

ART. **81**, *p.* 95.

4. $2\frac{1}{2}$.

5. $\frac{3}{4}$.

6. $1\frac{3}{13}$.

7. $\frac{8}{21}$.

8. $1\frac{1}{12}$.

9. $\frac{13}{14}$.

10. 6.

11. $1\frac{67}{153}$.

12. $\frac{7}{500}$.

13. $\frac{2}{15}$.

14. $\frac{7}{37}$.

15. 48.

16. $\frac{3}{4}$.

17. 16.

18. $29\frac{145}{227}$.

19. $1\frac{1027}{2000}$.

20. $\frac{8}{145}$.

21. 2.

22. $\frac{12}{31}$.

23. $1\frac{1}{2}$.

24. $\frac{15}{22}$.

25. 45.

26. $\frac{3}{8}$.

27. $1\frac{4}{11}$.

28. $3\frac{11}{63}$.

29. $\frac{153}{560}$.

30. $1\frac{83}{152}$.

31. $1\frac{13}{212}$.

32. $14\frac{7}{25}$.

33. $\frac{64}{175}$.

DECIMAL FRACTIONS.

ART. **90**,
p. 108.
13. 3.8235.
14. 4.9091.
15. 1.625.
16. 1.9636.
17. 6.875.
18. 21.6563.
19. 1.0978.
21. 13.094.
22. 4.7848.

ADDITION OF DECIMALS.

ART. **92**,
p. 110.
8. 757.4994.
9. 578.1023.
10. 247.0709.
11. 11316.1808.
12. 431.6186.
13. 312.5119.
14. 456.5192.
15. 260.889.
16. 7.6832.
17. 70.1779.
18. 100.001.
19. 122.
20. 81.1027.
21. 296.5799.
22. 155.2976.
23. 41.6363.
24. 684.2371.
25. 231.3898.
26. 38.647.

SUBTRACTION OF DECIMALS.

ART. **94**,
p. 112.
18. 0.999.
19. 9.9997.
20. 5.9994.
21. 4.128.
22. 4.2197.
23. 6.5766.
24. 4.3219 *yds.*
25. $4.8986.
26. 11.673 *lbs.*
27. 73.25 *ft.*
28. 221.761 *yds.*
29. 76.112 *lbs.*
30. $25.453.

MULTIPLICATION OF DECIMALS.

ART. **96**,
p. 114.
10. [illegible]
11. 1,744.3913.
12. 23.8894.
13. 386.5576.
14. 8,361.32.
15. 3.0088.
16. .2475.
17. 30.1534.
18. 63.8406.
19. .0027.
20. 20.4905.
21. 84.5688.
22. .3737.
23. 412.5508.
24. .1197.
25. .5103.
26. 87.4894.
27. 1713.782.
33. 964.5215.
34. 72.5641.
35. 29,170.4199.
36. 3.3538.
37. 112,320.2047.
38. 410.5526.
39. 76.9122.
40. 488.3785.

DIVISION OF DECIMALS.

ART. **98**,
p. 116.
9. 381.4089.
10. 112.[illegible]
11. 356.1111.
12. 12.5.
13. .0016.
14. 1,485.6.
15. 15,221.0061.
16. 153.8462.
17. .0625.
18. 32.27.
19. .025.
20. .916.

REDUCTION

ART. 143, p. 153.

17. 583,200 *sq. in.*
18. 2,280 *grs.*
19. 303,300 *sec.*
20. 1,428,840 *min.*
21. 145,600 *oz.*
22. 121,806 *oz.*
23. 12,990 *ft.*
24. 53,604 *in.*
25. 230 *pts.*
26. 1,554$\frac{3}{4}$ *in.*
27. 625$\frac{1}{2}$ *in.*
28. 14 *gals.* 3 *qts.* 2$\frac{6}{17}$ *gi.*
29. 11′ 6$\frac{2}{3}$″.
30. 7 *cwt.* 1 *qr.* 2 *lbs.* 4$\frac{4}{11}$ *oz.*
31. 5 *da.*
32. 17*s.* 1$\frac{7}{9}$ *qrs.*
33. 3 *pks.* 1$\frac{7}{9}$ *pts.*
34. 80,000 *oz.*

ART. 144, p. 158.

28. 104.75*s.*
29. 109.375 *gals.*
30. 3,887.8485 *rds.*
31. 2,855.7′.
32. $\frac{212}{425}$.
33. $\frac{423}{2000}$.
34. $\frac{313}{356}$.
35. $\frac{2331}{3520}$.
36. $\frac{1010}{3263}$.
37. $\frac{11}{19}$.
38. $\frac{71}{462}$.
39. $\frac{1}{6}$.
40. $\frac{171}{8960}$.

Miscellaneous Examples. p. 160.

21. £.8906.
22. $\frac{23}{270}$.
23. 1.7526 *lbs. Troy.*
24. .5144 *wks.*
25. $\frac{17}{63}$.
26. $\frac{103}{440}$.
27. $\frac{15}{32}$.
28. $\frac{23}{80}$.
29. 3*s.* 8*d.* 3.52*qrs.*
30. 4 *fur.* 23 *rds.* 11 *ft.* 2.64 *in.*

COMPOUND ADDITION.

ART. 146, pp. 162–4.

4. 83 *bu.* 6 *qts.*
5. 2 *bu.* 2 *pks.* 2 *qts.* 1 *pt.*
6. 27 *gals.* 2 *qts.* 1 *pt.*
7. 16 *yds.* 1 *ft.* 5 *in.*
8. 19 *A.* 3 *sq. ch.* 1926 *sq. li.*
9. 19 *da.* 23 *hrs.* 23 *min.*
10. £35 15*s.*
11. 107 *lbs.* 4 *oz.* 10 *dwts.*
12. 12 *mi.* 306 *rds.*
13. £92 0*s.* 1*d.*
14. 158 *bu.* 4 *qts.*
16. £1.8.
17. 5.0987 *da.*
18. £49 2*s.* 2$\frac{1}{2}$*d.*
19. 2 *lbs.* 6 *oz.* 19 *dwts.* 17 *grs.*
20. 1 ℔. 3 ℥ 3 ʒ 2 ℈ 18 *grs.*
21. 1 *T.* 10 *cwt.* 2 *qrs.* 4 *lbs.*
22. 39 *wks.* 15 *hrs.*
23. 25 *mi.* 253 *rds.* 1 *yd.* 1 *ft.* 6 *in.*
24. 59 *yds.* 2 *na.*
25. 221 *bu.* 2 *pks.* 5 *qts.*

COMPOUND SUBTRACTION.

ART. 148, pp. 167–8.

4. 2 *cwt.* 1 *qr.* 23 *lbs.*
5. 36 *hhds.* 49 *gals.*
6. 2 *yds.* 2 *in.*
7. 4 *A.* 145 *sq. rds.*
8. 15° 55′ 57″.
9. 17 ℔. 8 ℥ 4 ʒ 2 ℈ 16 *grs.*
17. 6° 17′ 25″.
18. £1.125 = £1 2*s.* 6*d.*
19. $\frac{129}{490}$ *hhd.* = 16 *gals.* 2 *qts.* 2$\frac{26}{35}$ *gi.*
20. 2,775.8 *grammes.*
21. 3 *da.* 22 *hrs.* 43 *min.* 34 *sec.*
22. 3 *T.* 16 *cwt.* 2 *qrs.* 15 *lbs.*
23. 5 *lbs.* 7 *dwts.* 13 *grs.*
24. 684 *gals.* 2 *qts.* 1 *pt.*
25. 15 *bu.* 1 *pk.* 3 *qts.*

COMPOUND MULTIPLICATION.

ART. **150**, *pp. 171–2.*

8. 1 *T.* 19 *cwt.* 2 *qrs.* 12 *lbs.*
9. $214.37½.
10. 3,006.1 *fr.*
11. 70.08 *kilog.*
12. 1,848.75 *meters.*
13. 322 *yds.*
14. 5 *hhds.*
15. 83 *wks.* 4 *da.*
16. 244 *lbs.* 4 *oz.*
17. 124 *yds.*
18. £10 3*s.* 4*d.*
19. 112 *A.* 120 *sq. rds.*
20. 3 *da.* 6 *hrs.* 4 *min.*
21. 134 *cwt.* 1 *qr.* 15 *lbs.*
22. £123 9*s.* 6*d.*
23. 865 *T.* 11 *cwt.* 3 *qrs.* 20 *lbs.*
24. £2,283 2*s.* 2¼*d.*
25. £4,808 14*s.*

COMPOUND DIVISION.

ART. **152**, *pp. 174–6.*

5. 12 *rds.* 11 *yds.* 2 *ft.* 10 *in.*
6. 20 *T.* 16 *cwt.* 1 *qr.* 1 *lb.* 4 *oz.*
12. 1 *lb.* 3 *oz.* 6 *dwts.*
13. £1 8*s.* 9*d.*
14. 5.2 *kilogrammes.*
15. 50.
16. 704.
17. 54.
18. 48 *mi.* 201 *rds.*
19. 1.42 *fr.*
20. 35 *liters.*
21. 38 *lbs.* 6 *oz.* 17 *dwts.*
22. £1 6*s.* [illegible].
23. 49 *qts.* [illegible].
24. 2 *mi.* 4 *fur.* 1 *rd.*
25. 48 *mi.* 201 *rds.*
26. 4 *sq. ft.* 54 *sq. in.*
27. 2 *lbs.* 3 *oz.* 17 *dwts.* 9 *grs.* nearly.
28. 1,500.
29. 7,040.
30. 1 *fr.* 42 *cent.*
31. 4 *hectog.* 2 *decag.*

Problems.

pp. 176–8.

4. 29 *A.* 96 *sq. rds.*
5. 48 *lbs.* 10 *oz.* 17 *dwts.* 4 *grs.*
6. 41 *A.* 140 *sq. rds.*
7. 1,288 *mi.* 280 *rds.*
8. 3 *lbs.* 3 *oz.* 17 *dwts.*
9. 54.
10. 1,920.
11. 9*s.* 5⅘*d.*
12. 1 *C.* 22 *cu. ft.* 1,486⅔ *cu. in.*
13. £1 12*s.* 2*d.*, and £19 6*s.*
14. 84 *fr.* 25 *cent.*
15. 6 *hrs.* 48 *min.*
16. 882 *fr.* 65 *cent.*
17. 1 *mi.* 1,180 *yds.*
18. 270 *yds.*
19. 1° 9′.
20. $23.65.

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2. Summ[illegible] of French Hi[illegible] **FRA**[illegible] A series of [illegible] three volu[illegible] [illegible] chronological [illegible] proportio[illegible]

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